# 3.8 Derivatives of Inverse Functions

If $f$ is differentiable at every point of an interval $I$ and $\dfrac{dy}{dx}$ is never zero on $I$, then $f$ has an inverse and $f^{-1}$ is differentiable at every point on the interval $f(I)$.

$$\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}(\arccos u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{arccsc} u) = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{arcsec} u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{arccot} u) = \frac{-1}{1+u^2}\frac{du}{dx}$$

Ex: $\frac{d}{dx}\left(\sin^{-1} x^2\right)$

$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$\boxed{\frac{2x}{\sqrt{1-x^4}}}$$

Ex: $\frac{d}{dt}\left(\tan^{-1}\sqrt{t}\right)$

$$\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{1}{1+(\sqrt{t})^2}\cdot\frac{1}{2}t^{-\frac{1}{2}}$$

$$\frac{1}{1+t}\cdot\frac{1}{2\sqrt{t}}$$

$$\boxed{\frac{1}{2\sqrt{t}\,(1+t)}}$$

Ex: Find $\frac{dy}{dx}$ of $y = \sec^{-1}(5x^4)$

$$\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{1}{|5x^4|\sqrt{(5x^4)^2-1}} \cdot 20x^3$$

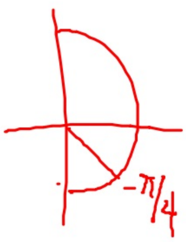
$$\frac{20x^3}{5x^4\sqrt{25x^8-1}} = \boxed{\frac{4}{x\sqrt{25x^8-1}}}$$

Ex: Find an equation for the line tangent to the graph of

$y = \cot^{-1}x$ @ $x = -1$.

$$m = \frac{d}{dx}\left(\cot^{-1}x\right) = \frac{-1}{1+x^2}\Bigg|_{x=-1} \quad \frac{-1}{1+(-1)^2} = \frac{-1}{2}$$

$$y = \cot^{-1}(-1) = \frac{\pi}{2} - \tan^{-1}(-1) = \frac{\pi}{2} + \left(+\frac{\pi}{4}\right) = \frac{3\pi}{4}$$




$$y - \frac{3\pi}{4} = -\frac{1}{2}(x+1)$$

Ex: $\frac{d}{dx}\left(\csc^{-1}\frac{x}{2}\right)$

How to simplify the radical:

$$\sqrt{\frac{x^2}{4}-1}$$

$$\sqrt{\frac{x^2}{4}-\frac{4}{4}}$$

$$\sqrt{\frac{1}{4}(x^2-4)}$$

$$\frac{1}{2}\sqrt{x^2-4}$$

$$\frac{-1}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2-1}}\cdot\frac{1}{2}$$

$$\frac{-1}{\frac{1}{2}|x|\sqrt{\frac{x^2}{4}-1}}\cdot\frac{1}{2}$$

$$\frac{-1}{|x|\sqrt{\frac{x^2}{4}-1}}=\frac{-1}{\frac{1}{2}|x|\sqrt{x^2-4}}=\frac{-2}{|x|\sqrt{x^2-4}}$$

Homework:

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