

3.9 Derivatives of Exponential and Logarithmic Functions

Derivative of e^x

If u is a differentiable function of x , then

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

Example: a. Find dy/dx if $y = e^{x^3}$

$$y' = e^{x^3} \cdot 3x^2$$

$$y' = 3x^2 e^{x^3}$$

b.) $y = e^{x^4 + x^2}$

$$y' = e^{x^4 + x^2} \cdot (4x^3 + 2x)$$

$$= (4x^3 + 2x)e^{x^4 + x^2}$$

c. $y = e^{\sin x}$

$$y' = e^{\sin x} \cos x$$

$$= \cos x e^{\sin x}$$

Derivative of a^x

If u is a differentiable function of x and for $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$$

Example: Find dy/dx if $y = 3^x$

$$y' = 3^x \ln 3$$

b. $y = 3^{2x}$

$$y' = 3^{2x} \ln 3 \cdot 2$$
$$= 2(3^{2x}) \ln 3$$

c.) $3^{\cot x}$

$$y' = 3^{\cot x} \ln 3 (-\csc^2 x)$$
$$= -3^{\cot x} (\ln 3) \csc^2 x$$

Derivative of $\ln x$

If u is a differentiable function of x and $u > 0$,

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

Example: Find $\frac{dy}{dx}$ if $y = \ln(3x^2)$

$$y' = \frac{1}{3x^2} \cdot 6x$$
$$= \frac{2}{x}$$

b.) $y = (\ln x)^2$

$$y' = 2 \ln x \cdot \frac{1}{x}$$
$$= \frac{2 \ln x}{x}$$

c.) $x \ln x - x$

$$y' = x \left(\frac{1}{x} \right) + \ln x (1) - 1$$

$$= 1 + \ln x - 1$$

$$y' = \ln x$$

Derivative of $\log_a x$

If u is a differentiable function of x and $u > 0$,

$$\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \frac{du}{dx}$$

Example: $y = \log_5 \sqrt{x}$

$$y' = \frac{1}{\sqrt{x} \ln 5} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2x \ln 5}$$

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#1-27 odd

$$x(t) = \sin^{-1}\left(\frac{t}{4}\right) \quad @ \quad t=3$$

$$v(t) = \frac{1}{\sqrt{1 - \left(\frac{t}{4}\right)^2}} \cdot \frac{1}{4}$$

$$= \frac{1}{4\sqrt{1 - \frac{t^2}{16}}} \Big|_{t=3}$$

$$\begin{aligned} \frac{1}{4\sqrt{1 - \frac{9}{16}}} &= \frac{1}{4\sqrt{\frac{7}{16}}} \\ &= \frac{1}{\cancel{4} \cdot \frac{\sqrt{7}}{\cancel{4}}} \\ &= \frac{1}{\sqrt{7}} \text{ OR } \frac{\sqrt{7}}{7} \end{aligned}$$