3.9 Cont.

Power Rule for Arbitrary Real Powers:

If u is a positive differentiable function of x and n is a differentiable function of x, and

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}.$$

Example:

a. Find y' if $y = x^{\sqrt{2}}$

$$y' = \sqrt{2}\, x^{\sqrt{2}-1}$$

b. $y = (2+\sin 3x)^{\pi}$

$$y' = \pi(2+\sin 3x)^{\pi-1} \cdot (\cos 3x)(3)$$

$$3\pi(2+\sin 3x)^{\pi-1}(\cos 3x)$$

Ex: If f(x) = ln (x-3), find f'(x) and state the domain of f'(x).

Find $f'(x)$:

$$f'(x) = \frac{1}{x-3}$$

Domain of $f(x)$:

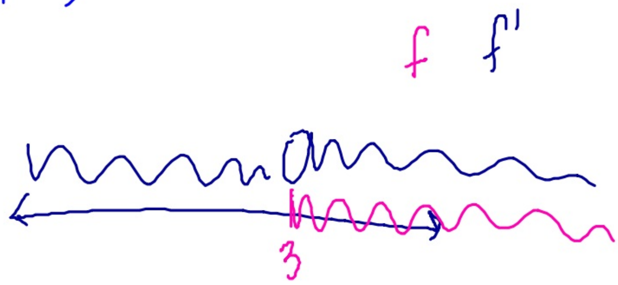
$x - 3 > 0$

$x > 3$

$(3, \infty)$

Domain of $f'(x)$

$(3, \infty)$



Logarithmic Differentiation:

Sometimes, the properties of logs can be used to simplify the differentiation process, even if we must introduce the logarithms ourselves as a step in the process.

Ex: Find dy/dx for $y = x^x$, for $x > 0$.

1. Take the ln of both sides

$$\ln y = \ln x^x$$

2. Prop. of logs

$$\ln y = x \ln x$$

3. Differentiate implicitly

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$$

$$\frac{dy}{dx} = y(1 + \ln x)$$

4. Substitute back in for y.

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

$$y = x^{\tan x}$$

$$\ln y = \ln x^{\tan x}$$

$$\ln y = \tan x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \tan x\left(\frac{1}{x}\right) + \ln x(\sec^2 x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\tan x}{x} + \ln x \sec^2 x$$

$$\frac{dy}{dx} = y\left(\frac{\tan x}{x} + \ln x \sec^2 x\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left(\frac{\tan x}{x} + \ln x \sec^2 x\right)$$

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