4.1 Extreme Values of Functions

One of the most useful things we can learn from a functions derivative is whether the function assumes any maximum or minimum values on a given interval and where these values are located if it does.

Absolute (Global) Extreme Values:

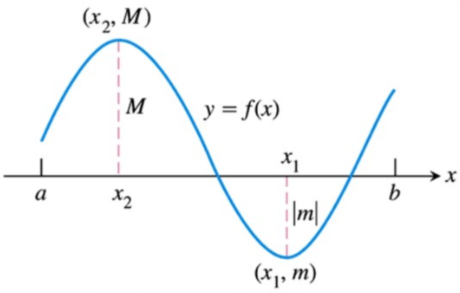
Let f be a function with domain D. Then f(c) is the

a.) absolute maximum value on D if and only if $f(x) \leq f(c)$ for all x in D.

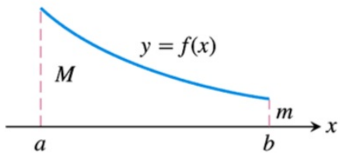
b.) absolute minimum value on D if and only if $f(x) \geq f(c)$ for all x in D.

The Extreme Value Theorem:

If f is continuous on a closed interval [a, b], then f has both a maximum value and a minimum value on the interval.

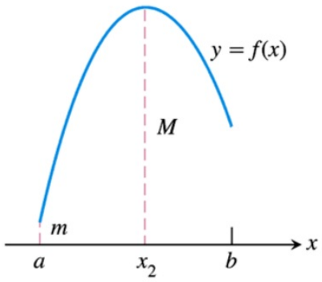


Maximum and minimum
at interior points

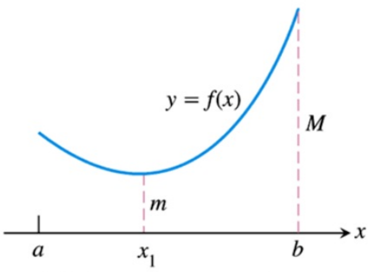


Maximum and minimum
at endpoints

*Note: The max and min values can occur at an endpoint OR an interior point!



Maximum at interior point,
minimum at endpoint



Minimum at interior point,
maximum at endpoint

Local (Relative) Extreme Values:

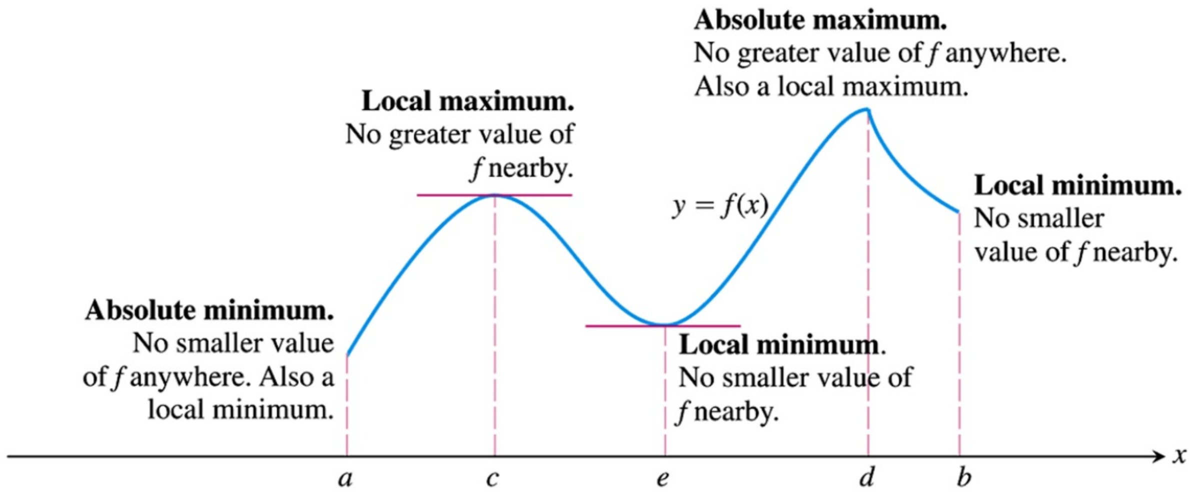
Let c be an interior point of the domain of the function f. Then f(c) is a

a.) local maximum value at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c.

b.) local minimum value at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c.

A function has a local maximum or local minimum at an endpoint c if the appropriate inequality holds for all x in some half-open domain interval containing c.

Classifying Extreme Values



$[a, b]$

Finding Extreme Values:

If a function f has a local max or min value at an interior point c of it's domain, and f' exists at c, then f'(c) = 0.

Because of this, we usually only need to look at a few points to find a function's extrema. These points would be on the interior domain where f' = 0 or f' does not exist and the domain endpoints. These possible points are called Critical Points (or critical numbers)!

Critical Point:
A point in the interior of the domain of a function f at which f' = 0 or f' does not exist.

Example: Find absolute minimum and maximum values of $f(x) = x^{2/3}$ on the interval [-2, 3].

Endpoints:

$f(-2) = (-2)^{2/3} = \left(\sqrt[3]{-2^2}\right) \approx 1.59$

$f(3) = (3)^{2/3} = \left(\sqrt[3]{3^2}\right) \approx 2.08$

Critical #s:

$f(0) = (0)^{2/3} = 0$

Find critical #s:

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

$f'(x) = \frac{2}{3\sqrt[3]{x}}$

Set numerator = 0 to find where f' is 0,
Set denominator=0 to find where f' is und.

f'(x) is never zero, but is undefined at x=0,
therefore 0 is a critical #.

Abs. max of $\sqrt[3]{9}$ at $x=3$.

Abs. min of 0 at $x=0$.

Example: Find the extreme values of $f(x) = \frac{1}{\sqrt{4-x^2}}$ on $(-2,2)$

Find critical #s:

$$f(x) = (4-x^2)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}(4-x^2)^{-\frac{3}{2}} \cdot (-2x)$$

$$f'(x) = \frac{x}{(4-x^2)^{3/2}}$$

f'(x) = 0 when x = 0 and
f'(x) is undefined at 2 & -2.
However, since 2 & -2 are not
included in the interval, they
are not part of the critical #s, only x=0 is.

$$f(0) = \frac{1}{\sqrt{4-0}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

.58 .5 .58

0

At x=0, we have a min of ½, which would also be the abs

p. 194 #11-18