4.2 Mean Value Theorem (MVT)

The MVT connects the average rate of change of a function over an interval with the instantaneous rate of change of the function at a point within the interval.

Mean Value Theorem for Derivatives:

If y = f(x) is continuous at every point of the closed interval [a, b] and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

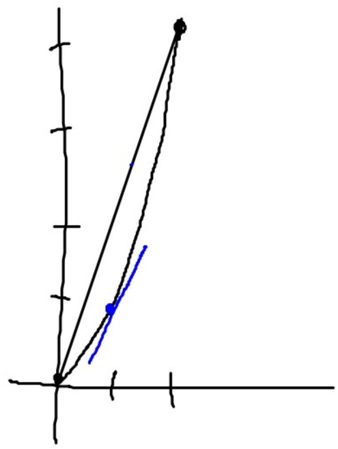
Example: Show that the function f(x) = x^2 satisfies the hypotheses of the MVT on the interval [0, 2]. Then find a solution c to the equation f'(c) = f(b) - f(a)/b-a.

$f'(x) = 2x$

$f'(c) = 2c$

Hypotheses:

1. $f(x) = x^2$ is cont. on $[0,2]$
2. $f(x) = x^2$ is diff on $(0,2)$



$$2c = \frac{f(2) - f(0)}{2-0}$$

$$2c = \frac{4-0}{2}$$

$$2c = 2$$

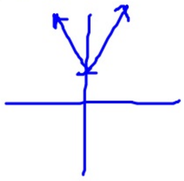
$$\boxed{c = 1}$$

Example: Explain why each of the following functions fails to satisfy the conditions of the MVT on the interval [-1, 1].

a. $f(x) = \sqrt{x^2} + 1$

$f(x) = |x| + 1$ has a corner at $x=0 \therefore f(x)$ is not diff at $x=0$.

$f(x) = |x| + 1$



b. $f(x) = \begin{cases} x^3 + 3 & for\ x < 1 \\ x^2 + 1 & for\ x \geq 1 \end{cases}$

$(1)^3 + 3 = 4$

$(1)^2 + 1 = 2$

$f(x)$ has a discont. @ $x=1$ $\therefore$ the MVT does not apply.

Homework:
p. 202 #1-8