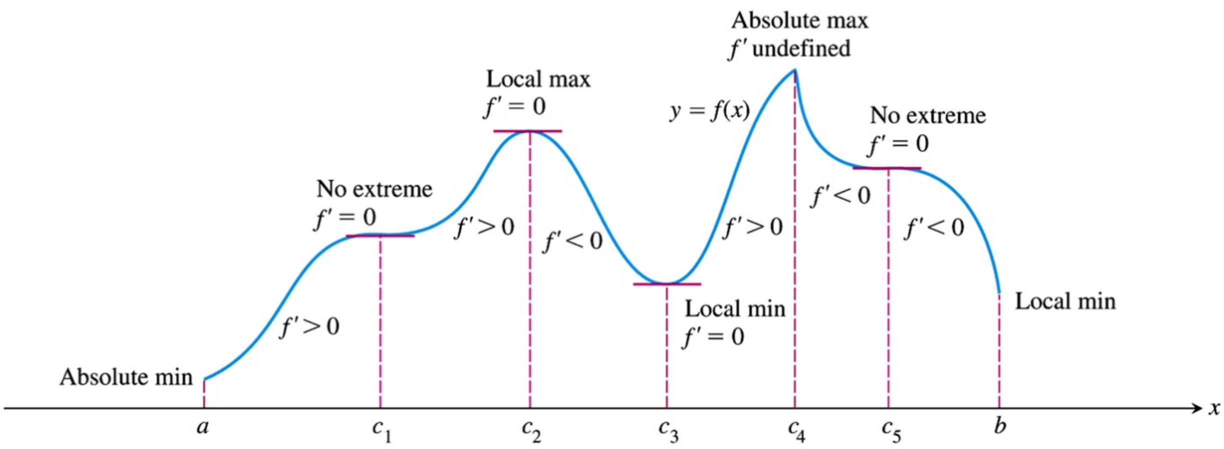
4.3 Connecting f' and f'' with the graph of f



FIRST DERIVATIVE TEST

The following test applies to a continuous function f(x).

At a critical point *c* :

1. If f' changes sign from positive to negative at c, then f has a local maximum value at c.
2. If f' changes sign from negative to positive at c, then f has a local minimum value at c.
3. If f' does not change sign at c, then f has no local extreme value at c.

At a left endpoint *a* :

If $f' < 0$ $(f' > 0)$ for $x > a$, then f has a local maximum (minimum) value at a.

At a right endpoint *b* :

If $f' < 0$ $(f' > 0)$ for $x < b$, then f has a local minimum (maximum) value at b.

Ex: Use the First Deriv. Test to find the local extreme values. Identify any absolute extrema.

a. $f(x) = x^3 - 12x - 5$

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2$

f' + | − | +
f inc −2 dec 2 inc

f(x) has a local max @ x = −2 because f'(x) changes from positive to negative @ x = −2.

f(x) has a local min @ x = 2 b/c f'(x) changes from neg to pos there

b. $f(x) = (x^2 - 3)e^x$

$f'(x) = (x^2-3)e^x + e^x(2x)$

$f'(x) = e^x(x^2-3) + 2xe^x$

$e^x(x^2-3) + 2xe^x = 0$

$e^x(x^2-3) = -2xe^x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1)$

$x = -3, 1$

f' + | − | +
f inc −3 dec 1 inc

Local max (at −3)

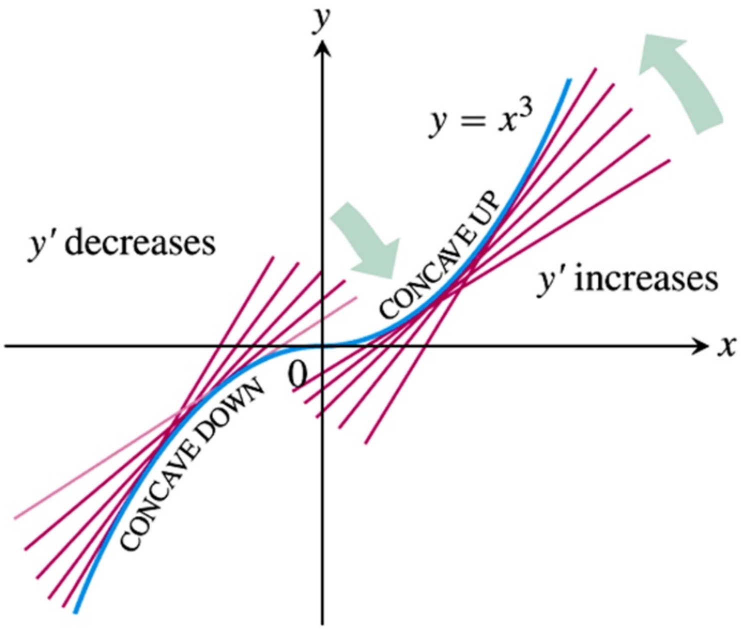
Abs. min (at 1)

CONCAVITY:

The graph of a differentiable function $y = f(x)$ is

(a) concave up on an open interval I if y' is increasing on I.

(b) concave down on an open interval I if y' is decreasing on I.



SECOND DERIVATIVE TEST:

The graph of a twice-differentiable function $y = f(x)$ is

(a) concave up on an open interval where $y'' > 0$.

(b) concave down on an open interval where $y'' < 0$.

POINTS OF INFLECTION

A point where the graph of a function has a tangent line and where the concavity changes is a **point of inflection.**

Example: Find all POI's of the graph of $y = e^{-x2}$

Example: Use the Concavity Test (Second Deriv. Test) to determine the concavity of the function over the given interval and the POIs:

a. $y = x^2$ on (3, 10)

$y' = 2x$

$y'' = 2$

$y'' > 0$ on $(3, 10)$ $\therefore$ $f(x)$ is concave ↑ on $(3, 10)$

NO POI's

b. $y = 3 + \sin x$ on $(0, 2\pi)$

$y' = \cos x$

$y'' = -\sin x$

f'': − on $(0, \pi)$, + on $(\pi, 2\pi)$

0 of conc ↓ π conc ↑ 2π

POI @ $x = \pi$

$f(x)$ is conc ↑ on $(\pi, 2\pi)$ b/c $y'' > 0$.

$f(x)$ is conc ↓ on $(0, \pi)$ b/c $y'' < 0$.

Can I Please Paint My Xylophone, My Xylophone Is Pink!!

f:	C	P	M	I
f':	I	M	X	P
f'':	P	X		

HW: p. 215 #1-19 odd