4.4 Modeling and Optimization

Strategy for Solving Max-Min Problems

1. Understand the Problem Read the problem carefully. Identify the information you need to solve the problem.

2. Develop a Mathematical Model of the Problem Draw pictures and label the parts that are important to the problem. Introduce a variable to represent the quantity to be maximized or minimized. Using that variable, write a function whose extreme value gives the information sought.

3. Graph the function Find the domain of the function. Determine what values of the variable make sense in the problem.

4. Identify the Critical Points and Endpoints Find where the derivative is zero or fails to exist.

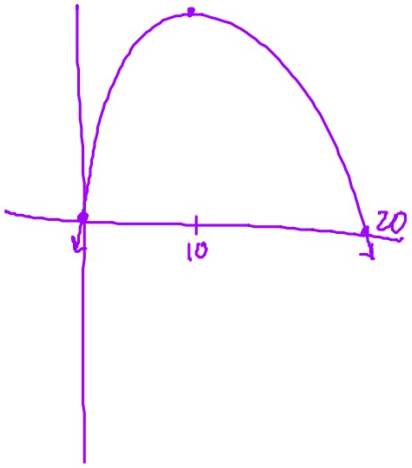
5. Solve the Mathematical Model If unsure of the result, support or confirm your solution with another method.

6. Interpret the Solution Translate your mathematical result into the problem setting and decide whether the result makes sense.

Ex: Find two numbers whose sum is 20 and whose product is as large as possible.

Let one # be x

2nd # be $20-x$



$f(x) = x(20-x) = 20x - x^2$

Graph - local max @ $x=10$

one # 10, the other # 10

$f'(x) = 20-2x$

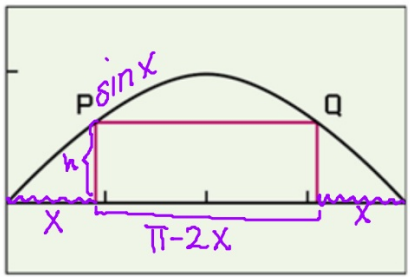
$20-2x=0$

$-2x=-20$

$x=10$

Ex 2: Inscribing Rectangles

A rectangle is to be inscribed under one arch of the sine curve as seen below. What is the largest area the rectangle can have and what dimensions give that area?



[0, π] by [–0.5, 1.5]

$A(x) = b \cdot h$ base $= \pi - 2x$ height $= \sin x$

$A(x) = (\pi - 2x)\sin x$

Graph - Locate Max ⇒

Max Area $= 1.12$ units^2

Base $= \pi - 2(.71) = 1.72$

Height $= \sin(.71) = .65$

$A'(x) = (\pi - 2x)\cos x + \sin x(-2)$

$(\pi - 2x)\cos x - 2\sin x = 0$

$x = .71$

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