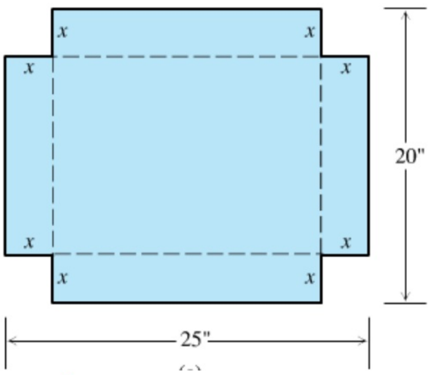
An open-top box is to be made by cutting congruent squares of side length x from the corners of a 20 by 25 inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting maximum volume?



$V = lwh$

$V = x(20-2x)(25-2x) = 4x^3 - 90x^2 + 500x$

$V' = 12x^2 - 180x + 500$

$x = \dfrac{180 \pm \sqrt{180^2 - 4(12)(500)}}{24} \approx \cancel{11.32},\ \boxed{3.68\text{in}}$

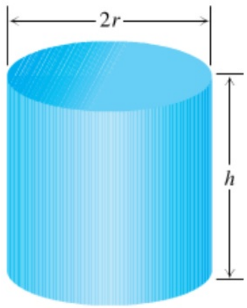
height $= x$

length $= 25-2x$

width $= 20-2x$

Max Volume: $V = 3.68(20-2\cdot3.68)(25-2\cdot3.68) \approx \boxed{820.53\text{in}^3}$

You have been asked to design a one-liter oil can shaped like a right circular cylinder. What dimensions will use the least material?



$$A = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$ Substitute in h

$$A = 2\pi r^2 + \frac{2000}{r} \longrightarrow 2\pi r^2 + 2000r^{-1}$$

Given:

$V = 1L$

$$\frac{\pi r^2 h}{\pi r^2} = \frac{1000\,cm^3}{\pi r^2}$$

$$h = \frac{1000}{\pi r^2}$$

$$A' = 4\pi r - 2000r^{-2}$$

$$= \frac{4\pi r^3}{r^2} - \frac{2000}{r^2}$$

$$A' = \frac{4\pi r^3 - 2000}{r^2}$$

$$4\pi r^3 - 2000 = 0$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \approx \boxed{5.42cm}$$

$$h = \frac{1000}{\pi(5.42)^2} \approx \boxed{10.84cm}$$

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