4.5 Linearization

In our study of the derivative, we have frequently referred to the tangent line to the curve. The tangent line is so important because it gives us a respresentation of the curve itself if we stay close enough to the point of tangency. We say differentiable curves are locally linear.

Linearization:

If f is differentiable at $x = a$, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a)$$

defines the linearization of f at a. The approximation $f(x) \approx L(x)$ is the standard linear approximation of f at a. The point $x = a$ is the center of the approximation.

Example: Find the linearization of f(x) = $\sqrt{1+x}$ at x = 0. Then use is to approximate $\sqrt{1.02}$ without a calculator. Use a calculator to determine the accuracy of the approximation.

Tangent Line:

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} = \frac{1}{2\sqrt{1+x}}$

$f'(0) = \frac{1}{2\sqrt{1+0}} = \frac{1}{2}$

$f(0) = \sqrt{1+0} = 1$

$y - 1 = \frac{1}{2}(x-0)$

$y = \frac{x}{2} + 1$

$\boxed{L(x) = \frac{x}{2} + 1}$

$\sqrt{1.02} \approx L(.02) = \frac{.02}{2} + 1 = .01 + 1 = \boxed{1.01}$

Actual (calc) $\sqrt{1.02} = 1.009950494$

Error: $|1.009950494 - 1.01| = 4.95 \times 10^{-5}$

$\boxed{\text{Error less than } 10^{-4}}$

<u>Example</u>: Find the linearization of f(x) = cos x at x = $\pi/2$. Then use is to approximate cos 1.75 without a calculator. Use a calculator to determine the accuracy of the approximation.

$f'(x) = -\sin x$

$f'(\pi/2) = -\sin(\pi/2) = -1$

$f(\pi/2) = \cos(\pi/2) = 0$

$y - 0 = -1(x - \pi/2)$

$\boxed{L(x) = -x + \frac{\pi}{2}}$

$\cos 1.75 \approx L(1.75) = \boxed{-1.75 + \frac{\pi}{2} = -.1792036732}$

Actual: $\cos 1.75 = -.1782460556$

Error: 9.58×10^{-4}

$\boxed{\text{Error less than } 10^{-3}}$

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