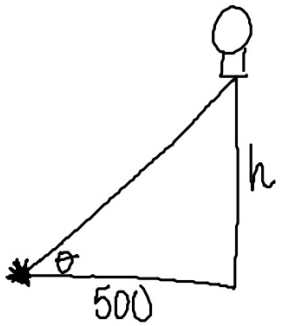
4.6 Related Rates, cont.

Ex: A hot-air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at a rate of 0.14 radians per minute. How fast is the balloon rising at that moment?



want: $\frac{dh}{dt}$

know: when $\theta = \pi/4$,

$\frac{d\theta}{dt} = .14$

$\tan\theta = \frac{h}{500}$

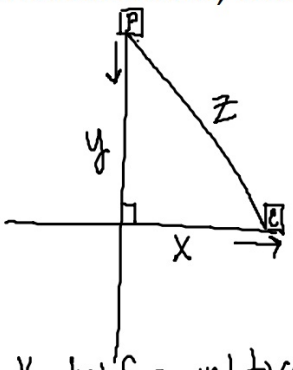
$h = 500\tan\theta$ ← Equation

$\frac{dh}{dt} = 500\sec^2\theta\frac{d\theta}{dt}$

$\frac{dh}{dt} = 500\sec^2(\pi/4)(.14)$

$\boxed{\frac{dh}{dt} = 140 \text{ ft/min}}$

Ex: A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now heading straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?



x = dist from int to car

y = dist from pol to int

z = dist b/w police & car

$z = \sqrt{.6^2 + .8^2} = 1$

Want: $\frac{dx}{dt}$

$x^2 + y^2 = z^2$

$x^2 = z^2 - y^2$ ← Equation

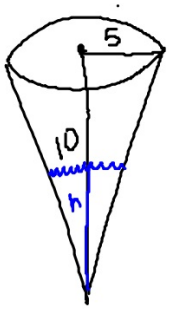
$2x\frac{dx}{dt} = 2z\frac{dz}{dt} - 2y\frac{dy}{dt}$

$x\frac{dx}{dt} = z\frac{dz}{dt} - y\frac{dy}{dt}$

$.8\frac{dx}{dt} = 1(20) - (.6)(-60)$

$\frac{dx}{dt} = 70$ mph

Ex: Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



$\frac{10}{5} = \frac{h}{r}$

$10r = 5h$

$r = \frac{h}{2}$

want: $\frac{dh}{dt}$

know: $\frac{dV}{dt} = 9$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{3}\left(\frac{h^2}{4}\right)h$

$V = \frac{\pi}{12}h^3$ ← Equation

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4}h^2 \frac{dh}{dt}$

$9 = \frac{\pi}{4}(6)^2 \frac{dh}{dt}$

$9 = 9\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{\pi} \approx$ .32 ft/min

HOMEWORK:
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