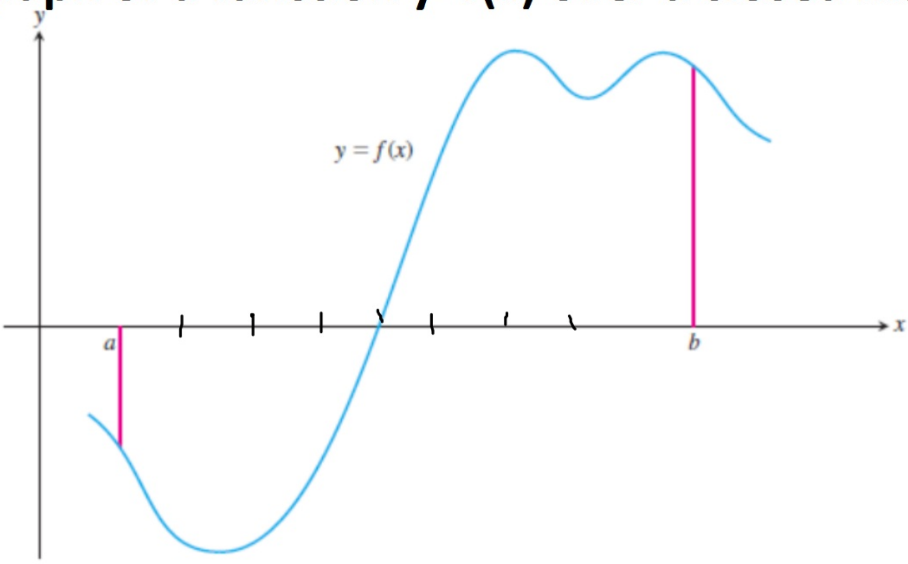
5.2 Definite Integrals

Riemann Sums:

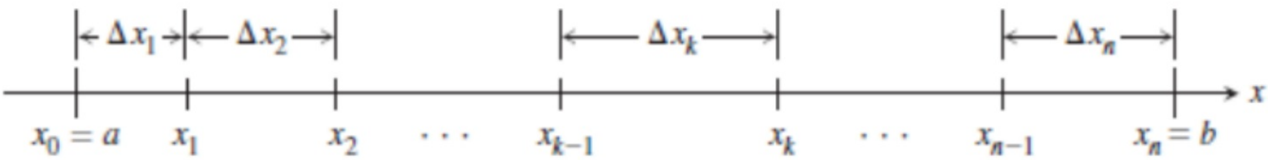
Sigma Notation Review

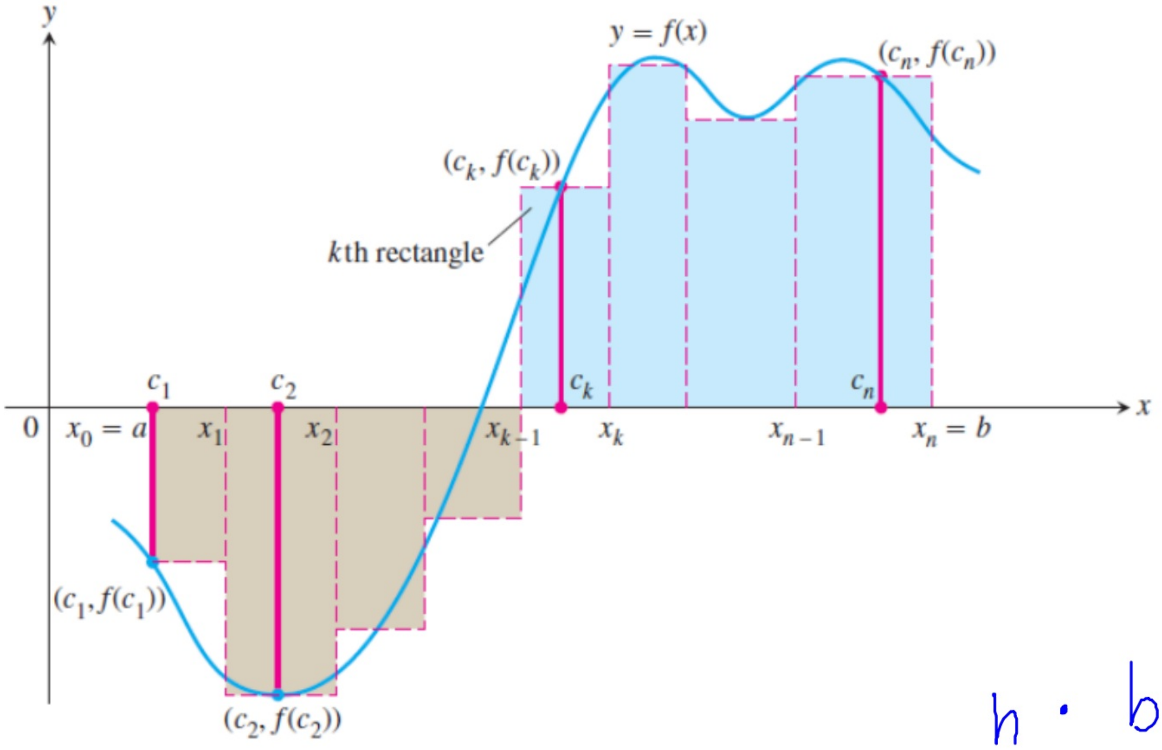
$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n.$$

The graph of a function y=f(x) over a closed interval [a,b]



Partitioned into subintervals of length $\Delta x_k = x_k - x_{k-1}$





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On each subinterval, we form the product $f(c_k) \cdot \Delta x_k$ (The area of each rectangle). We then take the sum of these products: $\sum_{k=1}^{n} f(c_k) \cdot \Delta x_k$. This is a Riemann sum for f on the interval [a,b].

As the partitions in [a,b] become finer and finer, our sum would become more accurate. Therefore, as the lengths of the subintervals tend to zero, the sums will converge to a common value. (THE LIMIT)

The DEFINITE INTEGRAL is a limit of Riemann sums.

THEOREM 1 The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

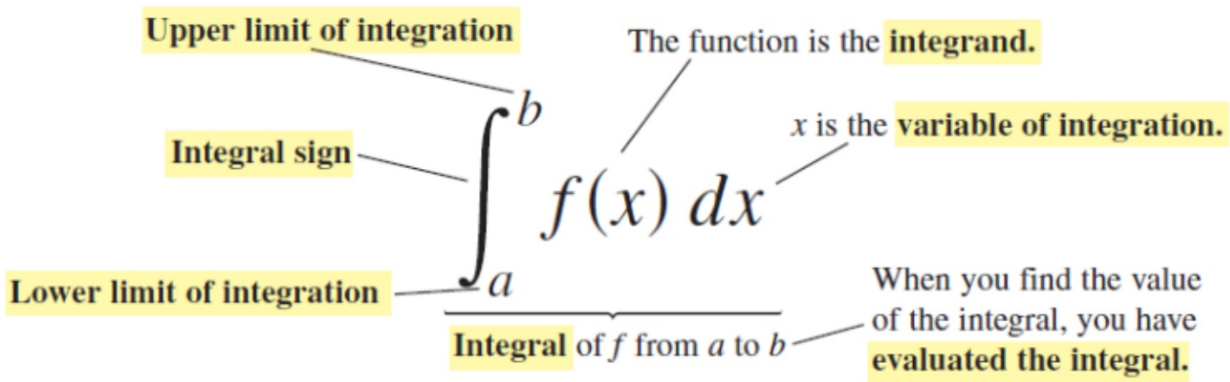
The Definite Integral of a Continuous Function on [a, b]

Let f be continuous on $[a, b]$, and let $[a, b]$ be partitioned into n subintervals of equal length $\Delta x = (b - a)/n$. Then the definite integral of f over $[a, b]$ is given by

$$\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k)\Delta x,$$

where each c_k is chosen arbitrarily in the k^{th} subinterval.

Other Notation:



"The integral from a to b of f(x)"

Example 1: The interval [-1, 3] is partitioned into n subintervals of equal length $\Delta x = 4/n$. Let m_k denote the midpoint of the k^{th} subinterval. Express the limit below as an integral.

$$\lim_{n \to \infty} \sum_{k=1}^{n} (3(m_k)^2 - 2m_k + 5)\Delta x$$

$$\int_{-1}^{3} (3x^2 - 2x + 5)\, dx$$

Ex 2: Express the following limit as an integral over the interval [-5, 3].

$$\lim_{n \to \infty} \sum_{k=1}^{n} c_k(3c_k - 2)^2 \Delta x$$

$$\int_{-5}^{3} x(3x-2)^2 \, dx$$

Assignment:
p. 282 #1-6