Area Under a Curve (as a Definite Integral)

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the area under the curve $y = f(x)$ from a to b is the **integral of f from a to b**, $A = \int_a^b f(x)dx$.

**When the region is a geometric shape, you can use an area formula to calculate the integral.

Ex: Evaluate the integral $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

Zeros:

$\sqrt{4-x^2} = 0$

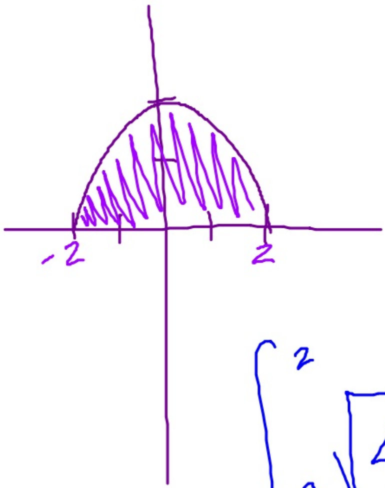
$4 - x^2 = 0$

$4 = x^2$

$x = \pm 2$

y-int:

$\sqrt{4-0^2} = \sqrt{4} = 2$

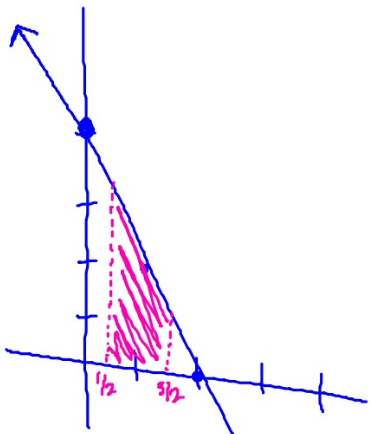


Area semi-circle:

$\frac{1}{2}\pi r^2$

$$\int_{-2}^{2} \sqrt{4-x^2}\, dx = \frac{1}{2}\pi(2)^2 = \boxed{2\pi}$$

Ex: Evaluate the integral $\int_{\frac{1}{2}}^{\frac{3}{2}} (-2x + 4)\, dx$



$f(1/2) = -2(1/2) + 4 = -1 + 4 = 3$

$f(3/2) = -2(3/2) + 4 = -3 + 4 = 1$

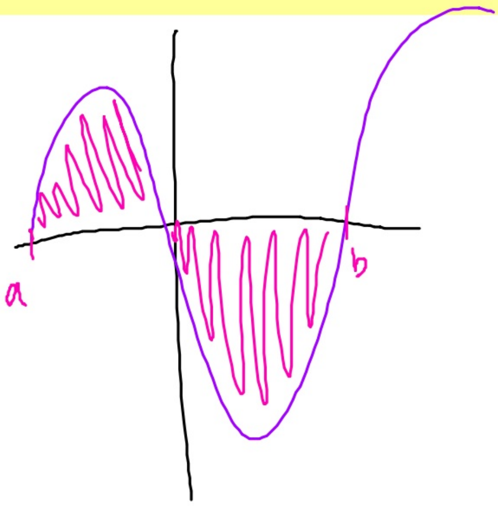
Area Trapezoid: $A = \frac{h}{2}(b_1 + b_2)$

$\int_{1/2}^{3/2} (-2x+4)\,dx = \frac{1}{2}(3+1) = \frac{1}{2}(4) = \boxed{2}$

Area

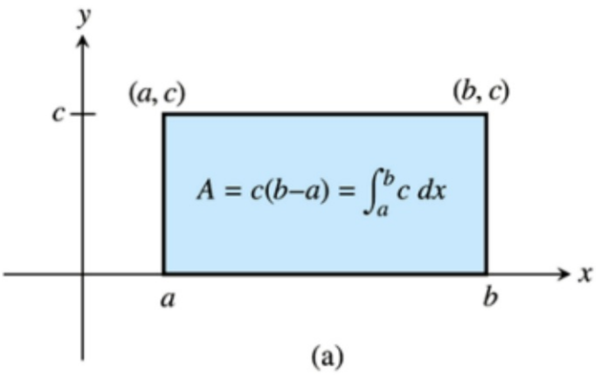
Area$= -\int_a^b f(x)dx$ when $f(x) \leq 0$.

$\int_a^b f(x)dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis})$.

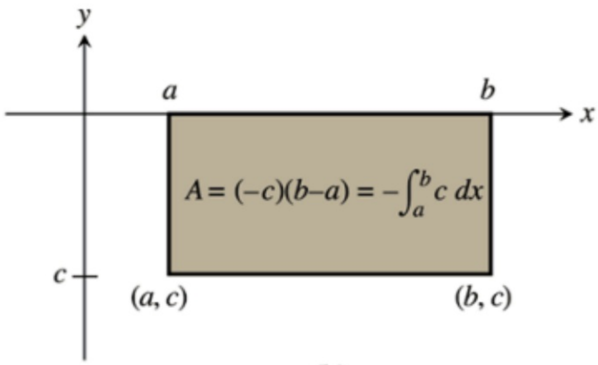


The Integral of a Constant

If $f(x) = c$, where c is a constant, on the interval $[a,b]$, then
$\int_a^b f(x)dx = \int_a^b c\,dx = c(b-a)$



(a)



(b)

Ex: A train moves along a track at a steady 75 mph from 7 to 9 am. Express its total distance traveled as an integral. Then evaluate the integral using Thm 2.

$$\int_7^9 75\,dx = 75(9-7)$$

$$= 150 \text{ mi.}$$

Assignment:
p. 282 #7-27 odd

And DON'T FORGET - BOOK CHECK BY WED!!