4.2 - Antiderivative

A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The process of finding an antiderivative is **antidifferentiation**.

For example: $F(x) = 2x^2$ $f(x) = 4x$

Ex. 1: Find all possible functions with the given derivative.

a. $f'(x) = 2x$

$F(x) = x^2 + C$ $\quad \frac{2x^2}{2}$

d. $f'(x) = e^x$

$F(x) = e^x + C$

b. $f'(x) = 3x^2$

$F(x) = x^3 + C$ $\quad \frac{3x^3}{3}$

e. $f'(x) = 1/(x+2)$

$F(x) = \ln(x+2) + C$

c. $f'(x) = 6x^2$

$F(x) = 2x^3 + C$ $\quad \frac{6x^3}{3}$

f. $f'(x) = 2x^{-3}$

$F(x) = \frac{2x^{-2}}{-2} = -x^{-2} + C$

Trick:

If $f'(x) = ax^n$, then $F(x) = \frac{ax^{n+1}}{n+1} + C$

Keep in mind this trick works only on polynomials

Try this one now:

$f'(x) = 3x^2 - 2x + 1$

$F(x) = \frac{3x^3}{3} - \frac{2x^2}{2} + 1x + C = x^3 - x^2 + x + C$

Ex: Find the function with the given derivative whose graph passes through the point P.

$f'(x) = \sin x \quad P(0, 2)$

$F(x) = -\cos x + 3$

$F(x) = -\cos x + C$

$2 = -\cos(0) + C$

$2 = -1 + C$

$C = 3$

Assignment:
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$$y = \frac{-x^2}{2}$$

$$\frac{-3}{\cancel{2}} = \frac{-x^2}{\cancel{2}}$$

$$\sqrt{3} = \sqrt{x^2}$$

$$x = \pm\sqrt{3}$$

$$x = \sqrt{3}$$

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

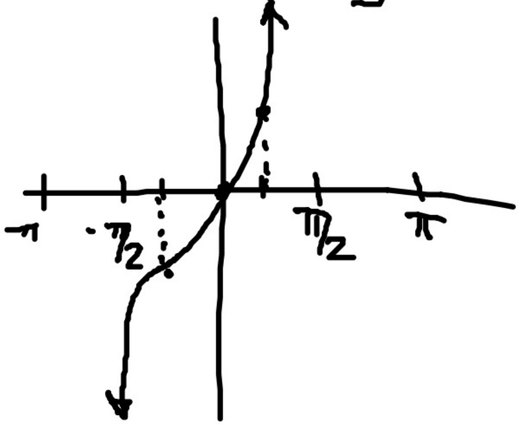
$$= \frac{1}{3-0}\int_0^3 \frac{-x^2}{2}\,dx$$

$$= \frac{1}{3}\left(-\frac{9}{2}\right)$$

$$= \frac{-3}{2}$$

$f(\theta) = \tan\theta$

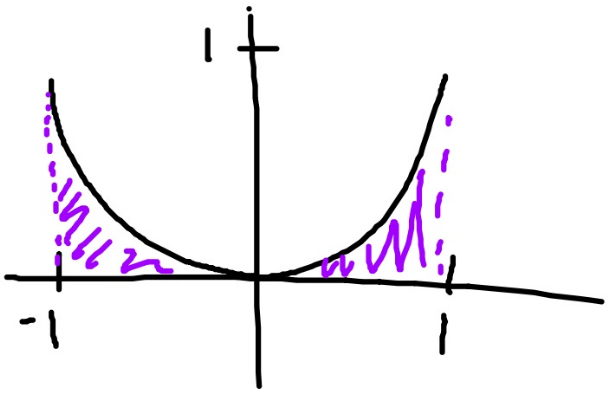
$\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$



$$\frac{1}{\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)} \int_{-\pi/4}^{\pi/4} \tan\theta$$

$$\frac{1}{-}\left(0\right) = 0$$

$f(t) = 1 - \sqrt{1-t^2} \quad [-1, 1]$



$2 - \frac{1}{2}\pi(1)^2$

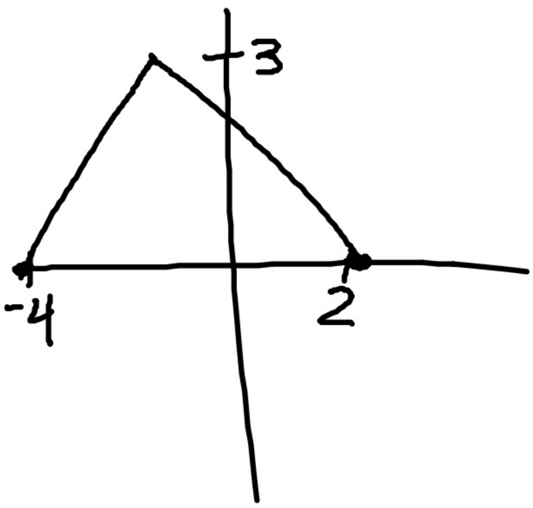
$2 - \frac{\pi}{2}$

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{1-(-1)}\int_{-1}^{1} 1 - \sqrt{1-t^2}\,dt$$

$$= \frac{1}{2}\left(2 - \frac{\pi}{2}\right)$$

$$= 1 - \pi/4 = \frac{4-\pi}{4}$$



$$\frac{1}{2-(-4)}\int_{-4}^{2} f(x)\,dx$$

$$\frac{1}{6}\left(\frac{1}{2}(6)(3)\right)$$

$$\frac{1}{6}(9)=\frac{9}{6}=\frac{3}{2}$$

$y = -3x^2 - 1 \quad [0, 1]$

$\frac{1}{1-0}\int_0^1 -3x^2 - 1\, dx$

$1(-2)$

$= -2$

$-2 = -3x^2 - 1$

$-1 = -3x^2$

$\sqrt{\frac{1}{3}} = \sqrt{x^2}$

$x = \pm\frac{1}{\sqrt{3}}$

$x = 1/\sqrt{3}$