5.3 Definite Integrals & Antiderivatives

Rules for Definite Integrals:

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ A definition

2. *Zero:* $\int_a^a f(x)\,dx = 0$ Also a definition

3. *Constant Multiple:* $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ Any number k

$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$ $k = -1$

4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:*

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \leq \int_a^b f(x)\,dx \leq \max f \cdot (b - a).$$

7. *Domination:* $f(x) \geq g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

$$f(x) \geq 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \geq 0 \qquad g = 0$$

Ex. Suppose $\int_{-1}^{1} f(x)dx = 5$, $\int_{1}^{4} f(x)dx = -2$, and $\int_{-1}^{1} h(x)dx = 7$

a. Find $\int_{4}^{1} f(x)dx$ if possible.

$$-\int_{1}^{4} f(x)dx = -(-2) = \boxed{2}$$

b. Find $\int_{-1}^{4} f(x)dx$ if possible.

$$\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx = 5 + (-2) = \boxed{3}$$

c. Find $\int_{-2}^{2} h(x)dx$ if possible.

Not possible

d. $\int_{-1}^{1} 4f(x) + h(x)\,dx$

$$4\int_{-1}^{1} f(x)dx + \int_{-1}^{1} h(x)dx$$

$$4(5) + 7$$

$$\boxed{27}$$

Ex: Show that the value of $\int_0^1 \sqrt{1+\cos x}\,dx$ is less than 3/2.

max f

$f(x) = \sqrt{1+\cos x}$

$f'(x) = \frac{1}{2}(1+\cos x)^{-\frac{1}{2}}(-\sin x)$

$= \dfrac{-\sin x}{2\sqrt{1+\cos x}}$

und

$1+\cos x = 0$

$\cos x = -1$

$x \neq \pi$

ZERO

$-\sin x = 0$

$x = 0$

$f(0) = \sqrt{1+\cos 0} = \sqrt{1+1} = \sqrt{2} \leftarrow$ max

$f(1) = \sqrt{1+\cos 1} = \sqrt{1.5}$

$$\int_0^1 \sqrt{1+\cos x}\,dx \leq \sqrt{2}(1-0)$$

$$\int_0^1 \sqrt{1+\cos x}\,dx \leq \sqrt{2}$$

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