5.3, cont.

Average (Mean) Value

If f is integrable on $[a,b]$, its average (mean) value on $[a,b]$ is

$$avg(f) = \frac{1}{b-a}\int_a^b f(x)dx$$

Find the average value of $f(x) = 2 - x^2$ on $[0,4]$.

$$\text{avg}(f) = \frac{1}{4-0}\int_0^4 (2-x^2)\,dx$$

$$= \frac{1}{4}\left(-\frac{40}{3}\right) = \boxed{\frac{-10}{3}}$$

Does f actually assume this value on the interval?

$$2 - x^2 = -\frac{10}{3}$$

$$-2 \qquad\qquad -2$$

$$-x^2 = \frac{-16}{3} \qquad x = \pm\frac{4}{\sqrt{3}}$$

$$x^2 = 16/3$$

yes - $x = 4/\sqrt{3}$

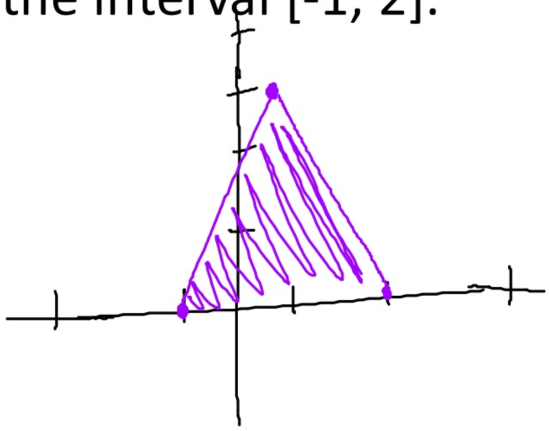
The Mean Value Theorem for Definite Integrals

If f is continuous on $[a,b]$, then at some point c in $[a,b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx.$$

Find the average value of f on the interval [-1, 2].

$$f(x) = \begin{cases} 2x + 2, & -1 \le x \le \frac{1}{2} \\ -2x + 4, & \frac{1}{2} \le x \le 2 \end{cases}$$



$$\text{avg}(f) = \frac{1}{2-(-1)} \int_{-1}^{2} f(x)\,dx$$

$$= \frac{1}{3}\left[\frac{1}{2}(3)(3)\right]$$

$$= \frac{1}{3}\left(\frac{9}{2}\right)$$

$$= \boxed{\frac{3}{2}}$$

P. 290
#11-18