5.4 Fundamental Theorem of Calculus

If f is continuous on $[a,b]$, then the function $F(x)=\int_a^x f(t)dt$ has a derivative at every point x in $[a,b]$, and

$$\frac{dF}{dt}=\frac{d}{dx}\int_a^x f(t)dt=f(x).$$

$$\frac{d}{dx}\left[F(x)-F(a)\right]$$

$$f(x)-0$$

The first part says that the definite integral of a continuous function is a differentiable function of its upper limit of integration. It also tells us what the derivative is. The second part says that the definite integral of a cont. function from a to b can be found from any one of the functions antiderivativ F as the number F(b) - F(a).

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

*This says that

1. Every continuous function f is the derivative of some other function
2. Every continuous function has an antiderivative
3. The processes of integration and differentiation are invereses of one another

Ex: Find $\frac{d}{dx}\int_{-\pi}^{x} \cos t \, dt$ using the FTC.

$\cos x$

$\frac{d}{dx}\left[\sin x - \sin(-\pi)\right]$

$\frac{d}{dx}(\sin x - 0)$

$\cos x - 0$

$$\frac{d}{dx}\int_{0}^{x} \frac{1}{1+t^2}\, dt$$

$$\frac{1}{1+x^2}$$

Ex: Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$

$\cos(x^2) \cdot 2x$

$2x\cos(x^2)$

$$y=\int_0^{3x} \frac{\sqrt{1+u^2}}{u}\,du \qquad \text{Find } \frac{dy}{dx}$$

$$\frac{\sqrt{1+(3x)^2}}{\cancel{3}x}\cdot \cancel{3}$$

$$\frac{\sqrt{1+9x^2}}{x}$$

Ex: Find $\frac{d}{dx}\int_x^5 3t\sin t\,dt$.

$$-\frac{d}{dx}\int_5^x 3t\sin t\,dt$$

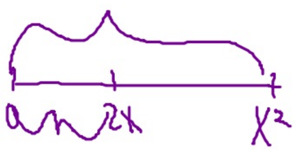
$$-3x\sin x$$

Find $\frac{d}{dx}\int_{2x}^{x^2} \frac{1}{2+e^t}\,dt$

$$\frac{d}{dx}\int_0^{x^2}\frac{1}{2+e^t}\,dt - \frac{d}{dx}\int_0^{2x}\frac{1}{2+e^t}\,dt$$

$$\frac{1}{2+e^{x^2}}\cdot 2x - \frac{1}{2+e^{2x}}\cdot 2$$

$$\frac{2x}{2+e^{x^2}} - \frac{2}{2+e^{2x}}$$



p. 302 #1-19 odd

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx \qquad F(x)=\tan^{-1}x$$

$$F(b)-F(a)$$

$$\tan^{-1}(1)-\tan^{-1}(-1)$$

$$\frac{\pi}{4}+\left(+\frac{\pi}{4}\right)$$

$$\frac{\pi}{2}$$

$y = \sin x \quad [0, \pi]$

$$av(f) = \frac{1}{\pi - 0} \int_0^{\pi} \sin x \, dx$$

$$\frac{1}{\pi}\left[-\cos(\pi) - (-\cos(0))\right]$$

$$\frac{1}{\pi}(1 + 1)$$

$$= 2/\pi$$

$y = \sec^2 x \quad [0, \frac{\pi}{4}]$

$$\frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \sec^2 x \, dx$$

$$\frac{4}{\pi}\left(\tan(\pi/4) - \tan(0)\right)$$

$$\frac{4}{\pi}(1 - 0) = \frac{4}{\pi}$$