The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a,b]$, and if F is any antiderivative of f on $[a,b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.
This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem**.

$$\int_a^b f(x)dx = F(b) - F(a)$$

Any definite integral of any continuous function f can be calculated without taking limits, without calculating Riemann sums, and often without effort - so long as an antiderivative of f can be found.

Ex: Evaluate $\int_{-1}^{3}(3x^2+1)\,dx$ using an antiderivative.

$$\int_{-1}^{3}(3x^2+1)dx = \left[\frac{3x^3}{3}+x\right]_{-1}^{3} = \left[(3)^3+3\right]-\left[(-1)^3+(-1)\right]$$

$$= 30-(-2)$$

$$= \boxed{32}$$

How to Find Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$ analytically,

1. partition $[a, b]$ with the zeros of f,
2. integrate f over each subinterval,
3. add the absolute values of the integrals.

How to find total area numerically (calculator)

To find the area between the graph of $y = f(x)$ and the x-axis over the interval $[a, b]$ numerically, evaluate

$\text{NINT}(|f(x)|, x, a, b)$

Integral = NET area (area above - area below)

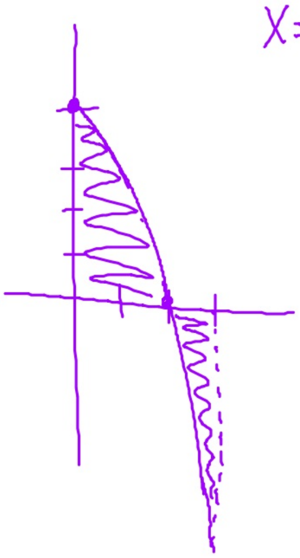
Area = TOTAL area

Ex: Find the area of the region between the curve $y = 4\text{-}x^2$ and the x-axis over the interval [0, 3]

Zeros: $4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$



$[0,2]: \int_0^2 (4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_0^2$

$\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4\cdot 0 - \frac{0^3}{3}\right) = \left(8 - \frac{8}{3}\right) - 0$

$= \frac{16}{3}$

$[2,3]\ \int_2^3 (4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_2^3$

$= \left(4\cdot 3 - \frac{3^3}{3}\right) - \left(4\cdot 2 - \frac{2^3}{3}\right) = 3 - \frac{16}{3} = \frac{-7}{3}$

Total area: $\left|\frac{16}{3}\right| + \left|-\frac{7}{3}\right| = \frac{16}{3} + \frac{7}{3} = \boxed{\frac{23}{3}}$

Ex: Find the area of the region between the curve $y = x\cos(2x)$ and the x-axis over the interval $[-3, 3]$ using your calculator.

≈ 5.425

Assignment

p. 302 #27-43 odd