5.5 Trapezoidal Rule

To approximate $\int_a^b f(x)dx$, use

$$T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n),$$

where $[a,b]$ is partitioned into n subintervals of equal length $h = (b - a)/n$.

Equivalently, $T = \dfrac{\text{LRAM}_n + \text{RRAM}_n}{2}$,

where LRAM_n and RRAM_n are the Rienamm sums using the left and right endpoints, respectively, for f for the partition.

Ex 1: Use the trapezoidal rule with n = 4 to estimate $\int_1^2 x^2\,dx$. Compare the estimate with the value of NINT and with the exact value. Then tell whether or not the estimate is an overestimate or underestimate.

Subintervals: $[1, \frac{5}{4}]$ $[\frac{5}{4}, \frac{3}{2}]$ $[\frac{3}{2}, \frac{7}{4}]$ $[\frac{7}{4}, 2]$ $h = \frac{1}{4}$

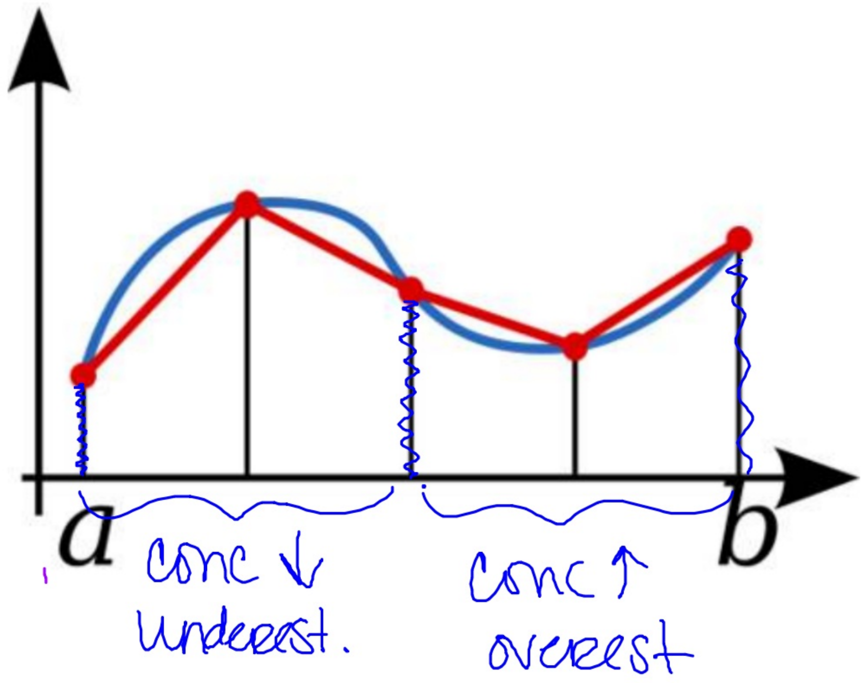
$$T = \frac{\frac{1}{4}}{2}\left[1 + 2\left(\frac{25}{16}\right) + 2\left(\frac{9}{4}\right) + 2\left(\frac{49}{16}\right) + 4\right]$$

$$= \frac{1}{8}\left[1 + \frac{25}{8} + \frac{9}{2} + \frac{49}{8} + 4\right]$$

$$= \frac{75}{32} \approx 2.34375$$

Exact: $\int_1^2 x^2\,dx = \left.\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

Assignment: P. 312 #1-8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