6.1 Slope Fields

An equation involving a derivative is called a differential equation. The order of a differential equation is the order of the highest derivative involved in the equation.

Ex: Solving a Differential Equation
Find all functions y that satisfy $dy/dx = \sec^2 x + 2x + 5$.

$y = \tan x + x^2 + 5x + C$

General Solution

First-order Differential Equation:

If the general solution to a first-order diff. eq. is continuous, the only additional information needed to find a unique solution is the value of the function at a single point, called an initial condition. A diff. eq. with an initial condition is called an initial-value problem. It has a unique solution, called the particular solution to the diff. eq.

Ex. 2: Solving an Initial Value Problem

Find the particular solution to the equation $dy/dx = e^x - 6x^2$ whose graph passes through the point (1, 0).

$y = e^x - 2x^3 + C$ ← General Sol'n

√ Cont.

$0 = e - 2(1)^3 + C$

$0 = e - 2 + C$

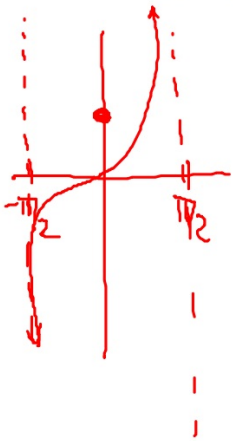
$C = 2 - e$

$y = e^x - 2x^3 + 2 - e$ ← Particular Sol'n

If the general solution is discontinuous, the initial condition only pins down the continuous piece of the curve that passes through the given point. It does not pin it down over the entire domain. In this case, the domain of the solution must be specified.

Ex. 3: Handling Discontinuity in an Initial Value Problem

Find the particular solution to the equation $dy/dx = 2x - \sec^2 x$ whose graph passes through the point (0, 3).



$y = x^2 - \tan x + C$ ← General, Not cont.

$3 = 0^2 - \tan 0 + C$

$3 = 0 + C$

$C = 3$

$y = x^2 - \tan x + 3$

$-\frac{\pi}{2} < x < \pi/2$ OR $(-\pi/2, \pi/2)$

When solving a diff. eq. where the anti-derivative isn't easily found, we can use the FTC.

Ex. 4: Using the FTC to Solve an Initial Value Problem
Find the solution to the diff. eq. $f'(x) = e^{-x2}$ for which $f(7)=3$.

$$y = \int_7^x e^{-t^2}\,dt + 3$$

Ex. 5: Graphing a General Solution

Graph the family of functions that solve the differential equation $dy/dx = \cos x$.

$y = \sin x + C$

$\{-10, -9, -8, -7, -6, \ldots 0, 1, 2, \ldots 10\} \rightarrow L_1$

Assignment
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