

6.2 Integration by Substitution

DEFINITION Indefinite Integral

The family of all antiderivatives of a function $f(x)$ is the **indefinite integral of f with respect to x** and is denoted by $\int f(x)dx$.

If F is any function such that $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$, where C is an arbitrary constant, called the **constant of integration**.

Properties of Indefinite Integrals

$$\int k f(x) dx = k \int f(x) dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

Power Formulas

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad \text{when } n \neq -1$$

$$\int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$$

(see Example 2)

Trigonometric Formulas

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \ln u du = u \ln u - u + C \quad (\text{See Example 2})$$

$$\int \log_a u du = \int \frac{\ln u}{\ln a} du = \frac{u \ln u - u}{\ln a} + C$$

Ex: Paying Attention to the Differential

Let $f(x) = x^3+1$ and let $u = x^2$. Find each of the following in terms of x .

$$a) \int f(x) dx = \int (x^3+1) dx = \frac{x^4}{4} + x + C$$

$$b) \int f(u) du = \int (u^3+1) du = \frac{u^4}{4} + u + C = \frac{x^8}{4} + x^2 + C$$

$$c) \int f(u) dx = \int (u^3+1) dx = \int (x^6+1) dx = \frac{x^7}{7} + x + C$$

Assignment
p. 337 #1-6, 13-16