6.2 Cont. - Integration by Substitution

When the integral is unfamiliar to us, often times a change of variables can turn it into one we can evaluate.

As seen yesterday, it is not sufficient to change an integral of the form $\int f(x)dx$ to an integral of the form $\int f(u)dx$. THE DIFFERENTIAL MATTERS!!! So we must do a complete substitution that changes the integral from $\int f(x)dx$ to $\int f(u)du$

Steps

1. **Determine what u will need to be.**
2. **Find the derivative of u (i.e. find du/dx)**
3. **Solve for du**
4. **Rewrite the integral in terms of u and substitute both u and du into your integral (manipulating it if needed).**
5. **Integrate**
6. **Replace u and you're done! :)**

Ex 4: Use the indicated substitution to evaluate the integral.

$\int \sin x \, e^{\cos x} \, dx$; *let* $u = \cos x$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$= -\int e^{\cos x} \cdot \underbrace{-\sin x \, dx}_{du}$

$= -\int e^{u} \, du$

$= -e^{u} + C$

$\boxed{= -e^{\cos x} + C}$

Ex 5: Use the indicated substitution to evaluate the integral.

$\int x^2\sqrt{5+2x^3}\,dx$; $let\ u = 5 + 2x^3$

$u = 5+2x^3$

$\frac{du}{dx} = 6x^2$

$du = 6x^2 dx$

$= \frac{1}{6}\int (5+2x^3)^{\frac{1}{2}} \cdot 6x^2 dx$

$= \frac{1}{6}\int u^{\frac{1}{2}}\, du$

$= \frac{1}{6}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{1}{9}u^{3/2} + C$

$\boxed{= \frac{1}{9}(5+2x^3)^{3/2} + C}$

Extra Ex: $\int x^2 e^{x^3}\,dx$; $\quad$ *let* $u = x^3$

$u = x^3$

$du = 3x^2\,dx$

$$= \frac{1}{3}\int e^{x^3} 3x^2\,dx$$

$$= \frac{1}{3}\int e^{u}\,du$$

$$= \frac{1}{3} \cdot e^{u} + C$$

$$\boxed{= \frac{1}{3} e^{x^3} + C}$$

Extra Ex: $\int 6x\sqrt{1+x^2}dx$; $let\ u = 1 + x^2$

$u = 1 + x^2$

$du = 2x\,dx$

$= 3\int (1+x^2)^{\frac{1}{2}} \cdot \frac{1}{3} \cdot 6x\,dx$

$= 3\int u^{\frac{1}{2}}\,du$

$= 3 \cdot \frac{2}{3} u^{\frac{3}{2}} + C$

$= 2u^{3/2} + C$

$\boxed{= 2(1+x^2)^{3/2} + C}$

Assignment
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