6.2 Day 4 - Substitution with Definite Integrals

Ex: Evaluate $\int_0^{\frac{\pi}{3}} \tan x \sec^2 x \, dx$

Let $u = \tan x$

$du = \sec^2 x \, dx$

$u(0) = \tan(0) = 0$

$u(\pi/3) = \tan(\pi/3) = \sqrt{3}$

$$= \int_0^{\sqrt{3}} u \, du$$

$$= \frac{u^2}{2}\Big]_0^{\sqrt{3}}$$

$$= \frac{(\sqrt{3})^2}{2} - \frac{0^2}{2} = \boxed{3/2}$$

Ex: Evaluate $\int_0^1 \frac{x}{x^2-4}\,dx$

$u = x^2 - 4$

$du = 2x\,dx$

$u(0) = 0^2 - 4 = -4$

$u(1) = 1^2 - 4 = -3$

$$= \frac{1}{2}\int_0^1 \frac{1}{x^2-4} \cdot 2x\,dx$$

$$= \frac{1}{2}\int_{-4}^{-3} \frac{1}{u}\,du$$

$$= \frac{1}{2}\left[\ln|u|\right]_{-4}^{-3}$$

$$= \frac{1}{2}\left(\ln|-3| - \ln|-4|\right)$$

$$= \frac{1}{2}(\ln 3 - \ln 4) = \boxed{\frac{1}{2}\ln\left(\frac{3}{4}\right)}$$

Assignment
p. 337 #53-65 odd