6.4 Separable Diff Eq/Exponential Growth & Decay

DEFINITION **Separable Differential Equation**

A differential equation of the form $dy/dx = f(y)g(x)$ is called **separable.** We **separate the variables** by writing it in the form

$$\frac{1}{f(y)}\, dy = g(x)\, dx.$$

The solution is found by antidifferentiating each side with respect to its thusly isolated variable.

Ex: Solve for y if dy/dx = $(xy)^2$ and y = 1 when x = 1.

$$\frac{dy}{dx} = x^2 y^2$$

$$\int \frac{1}{y^2}\,dy = \int x^2\,dx$$

$$-\frac{1}{y} = \frac{x^3}{3} + C$$

$$-1 = \frac{1}{3} + C$$

$$C = -\frac{4}{3}$$

$$-\frac{1}{y} = \frac{x^3}{3} - \frac{4}{3}$$

$$-\frac{1}{y} = \frac{x^3 - 4}{3}$$

$$\frac{1}{y} = \frac{4 - x^3}{3}$$

$$\boxed{y = \frac{3}{4 - x^3}}$$

Ex: Solve the initial value problem
dy/dx = (2x+1)(y+1) and y(-1) = 1

$$\int \frac{1}{y+1}\,dy = \int 2x+1\,dx$$

$$\ln(y+1) = x^2 + x + C$$

$$\ln 2 = C$$

$$\ln(y+1) = x^2 + x + \ln 2$$

$$e^{\ln(y+1)} = e^{x^2+x+\ln 2}$$

$$y+1 = e^{x^2+x} \cdot e^{\ln 2}$$

$$y+1 = 2e^{x^2+x}$$

$$\boxed{y = 2e^{x^2+x} - 1}$$

The Law of Exponential Change

If y changes at a rate proportional to the amount present (that is, if $dy/dt = ky$), and if $y = y_0$ when $t = 0$, then

$$y = y_0 e^{kt}.$$

The constant k is the **growth constant** if $k > 0$ or the **decay constant** if $k < 0$.

Ex: Find the solution to the diff. eq dy/dt = ky, where k is a constant, that satisfies the given conditions:

a. k = 2.3, y(0) = 75

$y = y_0 e^{kt}$

$y = 75e^{2.3t}$

b. y(0) = 90, y(10) = 45

$y = 90e^{kt}$

$45 = 90e^{10k}$

$\frac{1}{2} = e^{10k}$

$\ln \frac{1}{2} = \ln e^{10k}$

$-\ln 2 = 10k$ $\quad k = \frac{-\ln 2}{10}$

$\boxed{y = 90e^{\left(-\frac{\ln 2}{10}\right)t}}$

Today's Assignment

1. p. 357 #1-14

2. Go to our class website and print off the Chapter Review

3. 6.2 & 6.4 Quiz WEDNESDAY

4. TEST STILL FRIDAY 2/24