7.1 Integral as Net Change

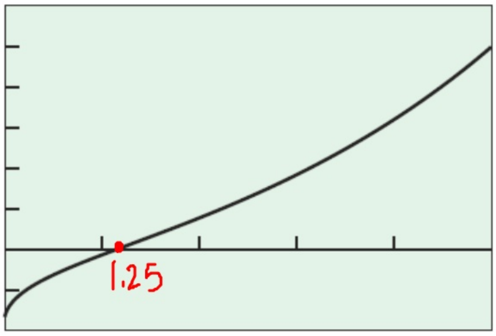
Recall from Ch. 5 that we could find the distance traveled by finding the area under the velocity curve.

Distance traveled = rate of change x time

We can apply this same logic to many other situations as well.

The integral is equal to the net change!

Ex 1: The graph of the given function shows the velocity of a particle moving along a horizontal s-axis for $0 \le t \le 5$. Describe the motion. $\frac{ds}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2}$ cm/sec



[0, 5] by [–10, 30]

$\frac{t^2(t+1)^2-8}{(t+1)^2}=0$

$t^2(t+1)^2=8$

$t(t+1)=2\sqrt{2}$

$t^2+t-2\sqrt{2}=0$

Quad Form: $t\approx 1.25$ $\cancel{t\approx -2\ldots}$

Moves left: $0<t<1.25$ $v(t)<0$

Moves right: $1.25<t<5$ $v(t)>0$

Stopped: $t=1.25$ $v(t)=0$

Speed Inc: $1.25<t<5$ $v(t)$ & $a(t)$ are same sign

Speed Dec: $0<t<1.25$ $v(t)$ & $a(t)$ diff signs

Displacement - the change in position (Δs). i.e. how far you end up from where you start.

Displacement = final position (s_f) - initial position (s_i) = $\int_i^f v(t)dt$

*This tells us that displacement is the NET AREA.

$\boxed{\text{Final Position } (s_f) = \text{initial position } (s_i) + \text{displacement}}$

Ex 2: Suppose the initial position of the particle in Ex 1 is s(0) = 9. What is the particle's position at (a) t = 1? (b) t = 5?

$s(1) = s(0) + \int_0^1 v(t)\,dt$

$s(1) = 9 + \int_0^1 \left[t^2 - \frac{8}{(t+1)^2}\right]dt = 9 + \int_0^1 t^2\,dt - 8\int_0^1 \frac{1}{(t+1)^2}\,dt$

$= 9 + \frac{t^3}{3}\Big]_0^1 - 8\left[\frac{-1}{t+1}\right]_0^1$

$u = t+1$
$du = 1\,dt$
$\int \frac{1}{u^2}\,du$
$\int u^{-2}\,du$
$-u^{-1}$

$$S(1) = 9 + \left.\frac{t^3}{3}\right]_0^1 - 8\left[\frac{-1}{t+1}\right]_0^1$$

$$= 9 + \left(\frac{1}{3} - 0\right) - 8\left(-\frac{1}{2} + 1\right)$$

$$= 9 + \left(\frac{1}{3} + 4 - 8\right)$$

$$= 9 + \left(-\frac{11}{3}\right) = \boxed{\frac{16}{3}}$$

$$S(5) = 9 + \int_0^5 \left[t^2 - \frac{8}{(t+1)^2}\right] dt$$

$$= 9 + \left.\frac{t^3}{3}\right]_0^5 - 8\left[\frac{-1}{t+1}\right]_0^5 = \boxed{44}$$

Total distance traveled = TOTAL AREA under v(t) = $\int |v(t)|\,dt$

Ex 3: Find the *total distance traveled* by the particle.

Split into subintervals

$$\left|\int_0^{1.25}\left[t^2-\frac{8}{(t+1)^2}\right]dt\right| + \left|\int_{1.25}^{5}\left[t^2-\frac{8}{(t+1)^2}\right]dt\right|$$

Calc: $\int_0^5 \left|t^2-\frac{8}{(t+1)^2}\right| dt \approx 42.59\text{ cm}$

Assignment

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