A car moving with initial velocity of 5 mph accelerates at the rate of $a(t) = 2.4t$ mph per second for 8 seconds.

(a) How fast is the car going when the 8 seconds are up?

(b) How far did the car travel during those 8 seconds?

$$\boxed{\text{Final Velocity} = \text{Initial Velocity} + \int a(t)\,dt}$$

KNOW

$v(0) = 5$ ←

$a(t) = 2.4t$

Method #1:

$$V(8) = V(0) + \int_0^8 2.4t\,dt$$

$$= 5 + 1.2t^2 \Big]_0^8$$

$$= 5 + (76.8 - 0)$$

$$V(8) = 81.8 \text{mph}$$

Method 2:

$v(t) = \int 2.4t$

$v(t) = 1.2t^2 + C$

$5 = 1.2(0)^2 + C$

$C = 5$

$v(t) = 1.2t^2 + 5$

$v(8) = 1.2(8)^2 + 5$

$= 81.8 \text{mph}$

b) $v(t) = 1.2t^2 + 5$

Zeros: $1.2t^2 = -5$

$t^2 = -5/1.2$

Distance traveled

$= \int_0^8 |v(t)|\,dt$

$= \int_0^8 (1.2t^2 + 5)\,dt$

$= \left[\frac{1.2t^3}{3} + 5t\right]_0^8 = 244.8 \text{ mph/sec} \Rightarrow \frac{\text{miles}}{\text{hr}} \cdot \text{sec}$

Conversion:

$244.8 \frac{\text{miles}}{\text{hr}} \cdot \text{sec} \left(\frac{1 \text{ hr}}{3600 \text{ sec}}\right)$

$\boxed{.068 \text{ miles}}$

Consumption Over Time

The integral is a natural tool to calculate net change and total accumulation of more quantities than just distance and velocity. Integrals can be used to calculate growth, decay, and, as in the next example, consumption. Whenever you want to find the cumulative effect of a varying rate of change, integrate it.

Ex:

From 1970 to 1980, the rate of potato consumption in a particular country was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1973?

$t=2$ $\quad$ $t=4$

1970: $t=0$

1980: $t=10$

$$\int_2^4 C(t)\,dt$$

$$\int_2^4 (2.2+1.1^t)\,dt \approx 7.066 \text{ millions of bushels}$$

$$\Downarrow$$

$$F(t) = 2.2t + \frac{1.1^t}{\ln 1.1}$$

A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for one hour as shown in Table 7.1. How many gallons were pumped during that hour?

Table 7.1 Pumping Rates

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

$$\int_0^{60} R(t)\,dt$$

$$\approx \frac{5}{2}\left[58+2(60)+2(65)+\cdots\cdots+2(63)+63\right]$$

$$\approx 3{,}582.5 \text{ gallons}$$

$$\frac{h}{2}(b_1+b_2)$$

P. 386
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$v(t) = \sqrt{4-t} \qquad 0 \le t \le 4$

Right $\sqrt{4-t} > 0$

Domain $4 - t \ge 0$

$-t \ge -4$

$t \le 4$

$0 \le t < 4$ Right

Left - never

Stopped: $4 - t = 0$

$t = 4$

Displacement

$$\int_0^4 \sqrt{4-t}\, dt = \int_0^4 (4-t)^{\frac{1}{2}}\, dt$$

$u = 4 - t \quad du = -1\, dt$

$u(0) = 4 \quad u(4) = 0$

$$= -\int_0^4 (4-t)^{1/2} \cdot -dt$$

$$= -\int_4^0 u^{1/2}\, du = \int_0^4 u^{1/2}\, du$$

$$= \left[\frac{2}{3} u^{3/2}\right]_0^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$$

$$\frac{16}{3}$$

$v(t) = 6\sin 3t \quad 0 \le t \le \frac{\pi}{2}$

Right

$6\sin 3t > 0$

$\sin 3t > 0$

sin

+ | +

$\frac{0}{3} < \frac{3t}{3} < \frac{\pi}{3}$

$0 < t < \pi/3$ Right

$\frac{\pi}{3} < t \le \pi/2$ Left

Stopped: $t = 0, \pi/3$

Disp:

$$\int_0^{\pi/2} v(t)\,dt = \int_0^{\pi/2} 6\sin 3t\,dt$$

$$6\int_0^{\pi/2} \sin 3t\,dt$$

Let $u = 3t$

$du = 3\,dt$

$u(0) = 0$

$u(\pi/2) = \frac{3\pi}{2}$

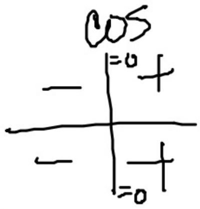
$$6\cdot\frac{1}{3}\int_0^{\pi/2} \sin 3t \cdot 3\,dt$$

$$2\int_0^{\frac{3\pi}{2}} \sin u\,du = 2\Big[-\cos u\Big]_0^{3\pi/2}$$

$$2\left[-\cos\frac{3\pi}{2} + \cos 0\right]$$

$$= 2(0+1) = 2$$

$v(t) = e^{\sin t}\cos t \qquad 0 \le t \le 2\pi$



Right

$\cos t > 0$

$0 \le t < \pi/2$

$\frac{3\pi}{2} < t \le 2\pi$

Left

$\cos t < 0$

$\frac{\pi}{2} < t < \frac{3\pi}{2}$

Stopped:

$\cos t = 0$

$t = \pi/2, \frac{3\pi}{2}$

$\int_0^{2\pi} e^{\sin t}\cos t \, dt$

$u = \sin t$
$du = \cos t \, dt$
$u(0) = 0$
$u(2\pi) = 0$

$\int_0^0 e^u \, du = \boxed{0}$

Final pos: $3 + 0 = 3$

C. $\int_0^{\pi/2} e^{\sin t}\cos t \, dt + \int_{3\pi/2}^{2\pi} e^{\sin t}\cos t \, dt + \int_{\pi/2}^{3\pi/2} e^{\sin t}\cos t \, dt$