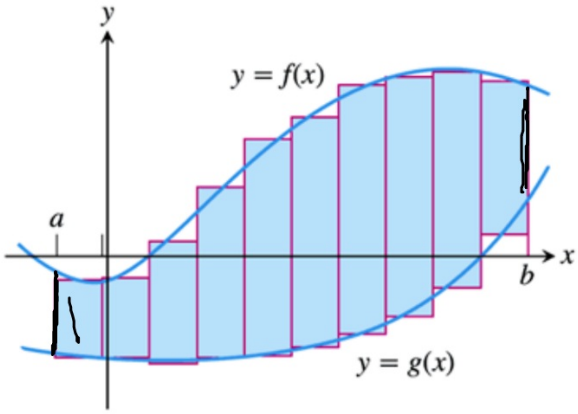
7.2 Areas in the Plane

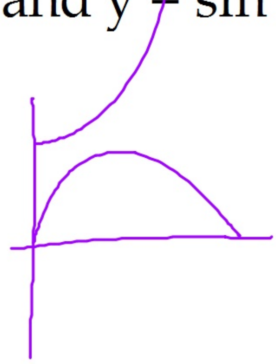
Area Between Curves:



If f and g are continuous with $f(x) \geq g(x)$ throughout $[a,b]$, then the area between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $[f - g]$ from a to b,

$$A = \int_a^b [f(x) - g(x)]\,dx$$

Ex: Find the area of the region between $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \pi$



$$A = \int_0^{\pi} (\sec^2 x - \sin x)\,dx$$

$$= \Big[\tan x + \cos x\Big]_0^{\pi}$$

$$= (\tan \pi + \cos \pi) - (\tan 0 + \cos 0)$$

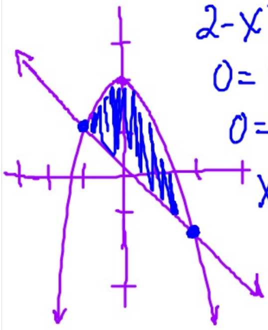
$$= 0 + (-1) - 0 - 1$$

$$\boxed{A = -2 \text{ units}^2}$$

Area Enclosed by Intersection of Curves

When finding area between intersecting curves, the limits of integration will come from the intersection points so we must calculate them first.

Ex: Find the area of the region enclosed by $y = 2 - x^2$ and $y = -x$.



$2 - x^2 = -x$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2, x = -1$

$$A = \int_{-1}^{2} \left[(2 - x^2) - (-x)\right] dx$$

$$= \int_{-1}^{2} (2 - x^2 + x)\, dx$$

$$= \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \left(4 - \frac{8}{3} + 2\right) - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$$

$$= 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \boxed{9/2 \text{ units}^2}$$

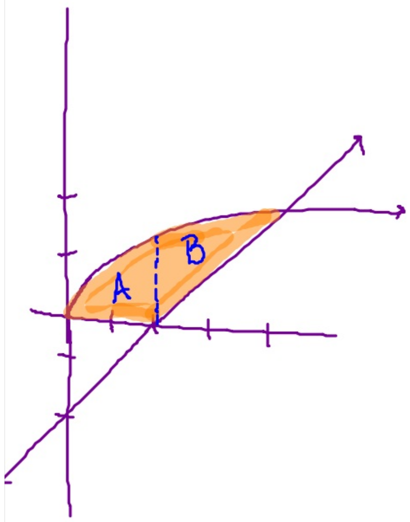
Ex: Using a calculator, find the area of the region enclosed by $y = 2\cos x$ and $y = x^2-1$.

$2\cos x = x^2-1$

fnInt($Y_1 - Y_2$, X, $-1.2\ldots$, $1.2\ldots$)

<u>Boundaries with Changing Functions</u>

Ex: Find the area of the enclosed region bounded above by $y = \sqrt{x}$ and below by the x-axis AND the line $y = x - 2$.



$A =$ Area of A + Area of B

$$A = \int_0^2 x^{\frac{1}{2}}\,dx + \int_2^4 \left[x^{\frac{1}{2}} - (x-2)\right]dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\Big]_0^2 + \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x\right]_2^4$$

$$= \left[\cancel{\frac{2}{3}(2)^{\frac{3}{2}}} - 0\right] + \left[\left(\frac{2}{3}\cdot 8 - \cancel{8} + \cancel{8}\right) - \left(\cancel{\frac{2}{3}(2)^{\frac{3}{2}}} - 2 + 4\right)\right]$$

$$= \frac{16}{3} - 2 = \boxed{\frac{10}{3}\text{ units}^2}$$

Assignment
p. 395 #1-10