7.2 Cont.

Integrating with Respect to y:

Sometimes the boundaries of a region are more easily described by functions of y than functions of x. For example, look at the graph of Ex 4 from yesterday. If we look at it from the perspective of horizontal rectangles, rather than vertical ones, we have a top function and bottom function and don't have to split it into separate regions.

When this is the case, we must first change the functions to y functions (i.e. solve them for x) and our limits of integration will also be y-values, not x-values. The "top" function would then be the "right" function and the "bottom" function would be the "left" function. The resulting integral would be of the form $A = \int_c^d [f(y) - g(y)]\, dy$

Ex: Find the area of the region in the prev. ex by integrating with respect to y.

$y=\sqrt{x} \Rightarrow x=y^2$

$y=x-2 \Rightarrow x=y+2$

$$A=\int_0^2 (y+2-y^2)\,dy$$

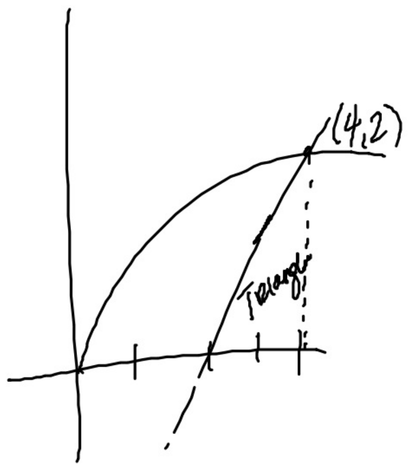
$$=\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_0^2$$

$$=\left(2+4-\frac{8}{3}\right)-0$$

$$=6-\frac{8}{3}=\frac{10}{3}u^2$$

We can also simplify the process by using geometry to save times, when it's applicable. Let's look at the same example again and see what else we could have done.

Ex: Use geometry to find the area of the region in Ex 4.



$$A = \int_0^4 x^{\frac{1}{2}}\,dx - \frac{1}{2}(2)(2)$$

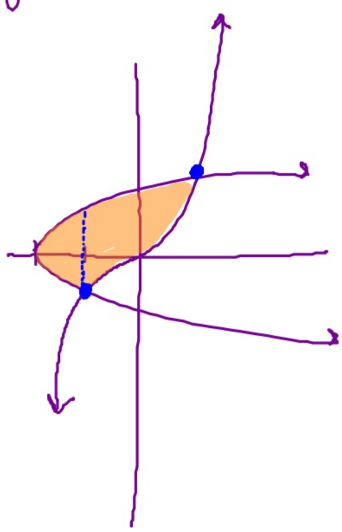
$$= \frac{2}{3}x^{3/2}\Big]_0^4 - 2$$

$$= \frac{16}{3} - 0 - 2$$

$$= 10/3\ u^2$$

Ex: Find the area of the region enclosed by the graphs $y = x^3$ and $x = y^2 - 2$.

$y = \pm\sqrt{x+2}$



Intersection:

$y^{\frac{1}{3}} = y^2 - 2$

$x = y^{\frac{1}{3}}$

$$A = \int_{-1}^{1.7930037} \left[y^{\frac{1}{3}} - (y^2 - 2) \right] dy$$

fnInt$\left(y^{\frac{1}{3}} - y^2 + 2, y, -1, 1.7930037 \right)$

Assignment

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