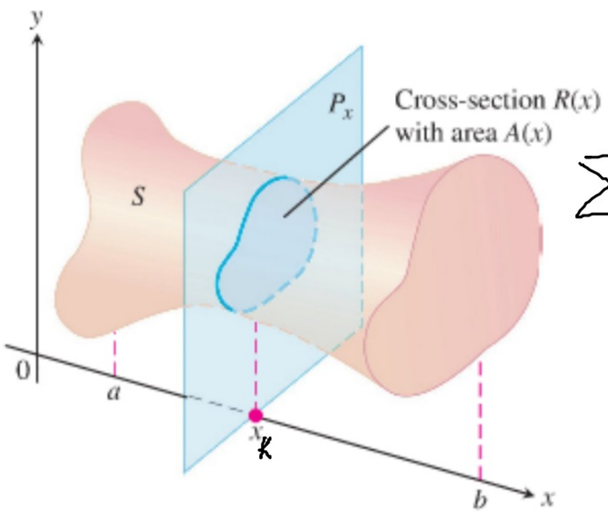
7.3 VOLUMES

Volume of a Solid

(with a known cross-section)

The definition of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b, $V = \int_a^b A(x)dx$.



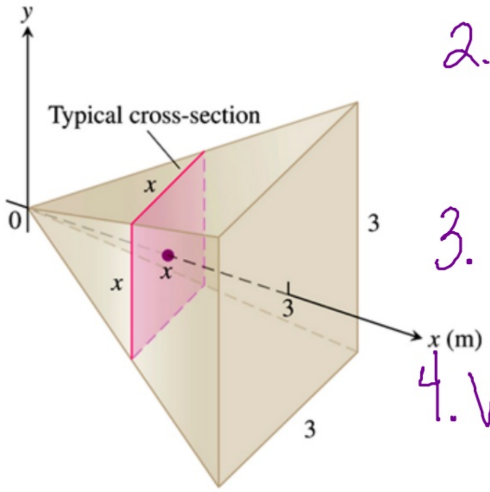
$V = \text{Area}_{\text{Base}} \times \text{Height}$

$\sum V = \sum A(x_k) \cdot \Delta x$

How to Find Volume by the Method of Slicing

1. Sketch the solid and a typical cross section.
2. Find a formula for $A(x)$.
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

Ex 1: A pyramid 3 m high has congruent triangular sides and a square base that is 3 m on each side. Each cross section of the pyramid parallel to the base is a square. Find the volume of the pyramid.



2. Cross-sections are squares $(A=s^2)$

$A(x) = x^2$

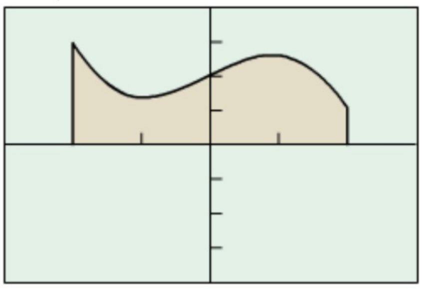
3. Limits of integration $x=0$ to $x=3$

4. $V=\int_0^3 x^2\,dx = \left.\frac{x^3}{3}\right]_0^3 = 9-0=\boxed{9\text{m}^3}$

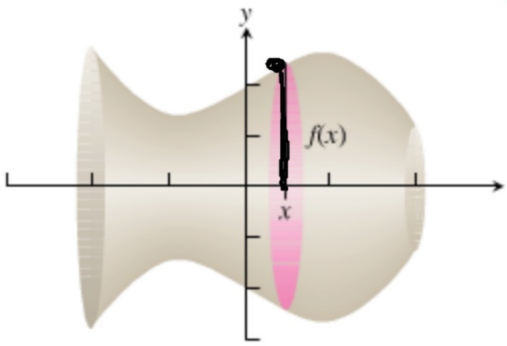
Ex 2 - A Solid of Revolution:

The region between the graph of $f(x) = 2+x\cos x$ and the x-axis over the interval [-2,2] is revolved about the x-axis to generate a solid. Find the volume of the solid.

1.



[–3, 3] by [–4, 4]



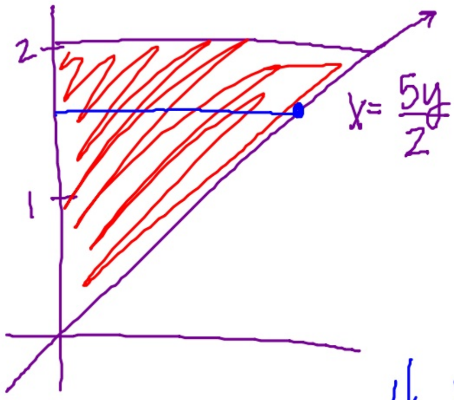
2. Cross-sections are circle ($A=\pi r^2$)

$$A(x)=\pi(2+x\cos x)^2$$

3. Limits of integration $x=-2$ to $x=2$

4. $$V=\int_{-2}^{2}\left[\pi(2+x\cos x)^2\right]dx \; \boxed{\approx 52.43 \text{ units}^3}$$

Find the volume of the solid generated by revolvng the shaded region about the y-axis.



2. cross-sections are circles

$$A(x) = \pi\left(\frac{5y}{2}\right)^2$$

3. Limits of integration $y=0$ to $y=2$

4. $V = \int_0^2 \left[\pi\left(\frac{5y}{2}\right)^2\right] dy = \int_0^2 \pi \cdot \frac{25y^2}{4}\, dy$

$= \frac{25\pi}{4}\int_0^2 y^2\, dy = \frac{25\pi}{4} \cdot \frac{y^3}{3}\Big]_0^2 = \frac{25\pi}{4}\left(\frac{8}{3} - 0\right)$

$= \boxed{\frac{50\pi}{3} \text{ units}^3}$

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