Given

$\frac{dV}{dt} = 100\pi$

Want

$\frac{dr}{dt}$? when $r = 5$

#11a)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$100\pi = 4\pi (5)^2 \frac{dr}{dt}$

$100\pi = 100\pi \frac{dr}{dt}$

$\frac{dr}{dt} = 1$ ft/min

11b

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi(5)(1)$$

$$\frac{dS}{dt} = 40\pi \text{ ft}^2/\text{min}$$

#17

Given

$\frac{dV}{dt} = -50$

Want

$\frac{dh}{dt}$ when $h = 5$

$\frac{h}{r} = \frac{6}{45}$

$\frac{45h}{6} = \frac{6r}{6}$

$r = 7.5h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (7.5h)^2 h$

$V = 18.75\pi h^3$

$\frac{dV}{dt} = 18.75\pi \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 56.25\pi h^2 \frac{dh}{dt}$

$-50 = 56.25\pi (5)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = -.0113$ m/min

or -1.13 cm/min

1.13 cm/min

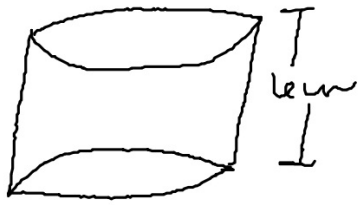
#17b) $r = 7.5h$

$$\frac{dr}{dt} = 7.5\frac{dh}{dt}$$

$$\frac{dr}{dt} = 7.5(-1.13)$$

$$\frac{dr}{dt} = -8.49 \text{ cm/min}$$

#15



Know

$\frac{dr}{dt} = \frac{.001}{3} = \frac{1}{3000}$

Want

$\frac{dV}{dt}$ when $d = 3.8$

$V = \pi r^2 h$

$V = \pi r^2 (6)$

$V = 6\pi r^2$

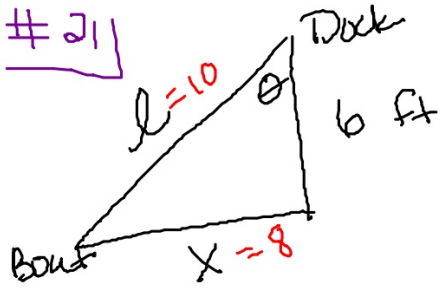
$\frac{dV}{dt} = 6\pi \cdot 2r \frac{dr}{dt}$

$\frac{dV}{dt} = 12\pi r \frac{dr}{dt}$

$\frac{dV}{dt} = 12\pi (1.9)\left(\frac{1}{3000}\right)$

$\frac{dV}{dt} = .024 \text{ in}^3/\text{min}$

#21



$$\frac{d\ell}{dt} = -2$$

want $\frac{dx}{dt}$ when $\ell = 10$

$$x^2 + 36 = \ell^2$$

$$x^2 = \ell^2 - 36$$

$$2x\frac{dx}{dt} = 2\ell\frac{d\ell}{dt}$$

$$2(8)\frac{dx}{dt} = 2(10)(-2)$$

$$\frac{dx}{dt} = \frac{-40}{16} = -2.5 \text{ ft/sec}$$

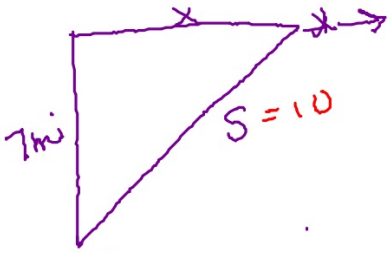
#16

$$\cos\theta = \frac{6}{\ell} \quad \| \quad \theta = \cos^{-1}\left(\frac{6}{\ell}\right)$$

$$-\sin\theta\frac{d\theta}{dt} = -\frac{6}{\ell^2}\frac{d\ell}{dt}$$

$$-(8/10)\frac{d\theta}{dt} = \frac{-6}{100}(-2)$$

$$\frac{d\theta}{dt} = -\frac{3}{20} \text{ or } -.15 \text{ rad/sec}$$



Given

$\frac{ds}{dt} = 300$

Want

$\frac{dx}{dt}$ when $s = 10$

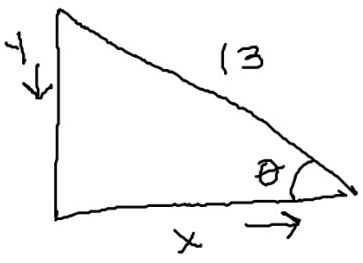
$$x^2 + 49 = s^2$$

$$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

$$2(\sqrt{51})\frac{dx}{dt} = 2(10)(300)$$

$$\frac{dx}{dt} = \frac{6000}{2\sqrt{51}} \approx 420.08 \text{ mph}$$

#19



Given

$\frac{dx}{dt} = 5$ ft/sec

Want

$\frac{dy}{dt}$

$x^2 + y^2 = 169$

$y^2 = 169 - x^2$

$2y \frac{dy}{dt} = -2x \frac{dx}{dt}$

$2(5)\frac{dy}{dt} = -2(12)(5)$

$\frac{dy}{dt} = \frac{-120}{10} = -12$ ft/sec

b]

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$

$\frac{dA}{dt} = \frac{1}{2}[12(-12) + 5(5)]$

$\frac{dA}{dt} = -\frac{119}{2}$ ft^2/sec

#12

$$\sin\theta = \frac{y}{13}$$

$$\cos\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dy}{dt}$$

$$\left(\frac{12}{13}\right)\frac{d\theta}{dt} = \frac{1}{13}(-12)$$

$$\frac{d\theta}{dt} = -1 \text{ rad/sec.}$$

#9

$\frac{dl}{dt} = -2$

$\frac{dw}{dt} = 2$

a) $A = lw$

$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$

$\frac{dA}{dt} = 12(2) + 5(-2)$

$\frac{dA}{dt} = 14 \text{ cm}^2/\text{sec}$

b) $P = 2l + 2w$

$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$\frac{dP}{dt} = 2(-2) + 2(2)$

$\frac{dP}{dt} = 0 \text{ cm/sec}$

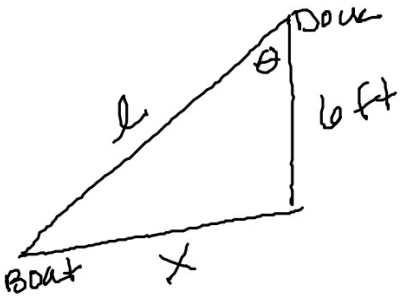
c)

$D^2 = l^2 + w^2$

$2D\frac{dD}{dt} = 2l\frac{dl}{dt} + 2w\frac{dw}{dt}$

$(13)\frac{dD}{dt} = 12(-2) + (5)(2)$

$\frac{dD}{dt} = \frac{-14}{13} \text{ cm/sec}$



Given

$\frac{dl}{dt} = -2$

Want

$\frac{dx}{dt}$ when $l = 10$

$x^2 + 6^2 = l^2$

$2x \frac{dx}{dt} = 2l \frac{dl}{dt}$

$2(8) \frac{dx}{dt} = 2(10)(-2)$

$\frac{dx}{dt} = \frac{-40}{16} = -\frac{5}{2}$ ft/sec

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b] $\tan\theta = \frac{x}{6}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{6} \frac{dx}{dt}$

$\left(\frac{5}{3}\right)^2 \frac{d\theta}{dt} = \frac{1}{6}\left(-\frac{5}{2}\right)$

$\frac{25}{9} \frac{d\theta}{dt} = \frac{-5}{4} \cdot \frac{9}{25}$ 3 5

$\frac{d\theta}{dt} = \frac{-3}{20}$ rad/sec