5.3, cont.

The need to calculate instantaneous rates of change led the discoverers of calculus (Newton & Leibnez) to an investigation of the slopes of tangent lines and ultimately, to the derivative --to what we call *differential calculus*.
But derivatives revealed only half the story. In addition to a calculation method (a "calculus") to describe how functions change at any given instant, they needed a method to describe how those instantaneous changes could accumulate over an interval to produce a function. This is why they also investigated areas under curves, which ultimately led to the second main branch of calculus --*integral calculus*.
Once Newton & Leibnez had the calculus for finding slopes of tangent lines and the calculus for finding areas under curves, the challenge for them was to prove the connection that they knew intuitively had to be there. The discovery of this connection (The Fundamental Theorem of Calculus) brought differential and integral calculus together to become the single most powerful insight mathematicians had ever acquired for understanding how the universe worked!!!

The Derivative of an Integral

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

So, what does this mean??

It means that the integral is an antiderivative of f!

$$\int_a^x f(t)dt = F(x) + C$$

$$\int_a^a f(t)dt = F(a) + C$$

$$0 = F(a) + C$$

$$C = -F(a)$$

$$\boxed{\int_a^x f(t)dt = F(x) - F(a)}$$

Example: Find the following integrals using the formula from the previous slide.

$$\int_a^x f(t)\,dt = F(x) - F(a)$$

a. $\int_0^\pi \sin x \, dx$

$F(x) = -\cos x$

$\int_0^\pi \sin x \, dx = F(\pi) - F(0) = -\cos\pi - (-\cos 0) =$

$= -(-1) - (-1) = 1 + 1 = \boxed{2}$

b. $\int_{-1}^{2} 3x^2 dx$

$F(x) = x^3$

$\int_{-1}^{2} 3x^2 dx = F(2) - F(-1) = (2)^3 - (-1)^3 = 8 - (-1) = \boxed{9}$

Example: Find the average value of the function on the interval, using antiderivatives to compute the integral.

a. $y = \frac{1}{x}$, $[e, 2e]$

$$\text{avg}(f) = \frac{1}{2e-e}\int_e^{2e} \frac{1}{x}\,dx$$

$$= \frac{1}{e}\left[\ln 2e - \ln e\right] = \frac{1}{e}\left(\ln 2 + \cancel{\ln e} - \cancel{\ln e}\right)$$

$$= \boxed{\frac{\ln 2}{e}}$$

$F(x) = \ln x$

Assignment
p. 290 #19-35 odd

5.3 Quiz Friday